Syllabus for POLI 784: Linear Methods in Causal Inference

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Course description

This is the second course in the PhD-level methods sequence of the Department of Political Science at UNC-Chapel Hill. Its objective is to provide students with a contemporary understanding of advancements in causal inference methods, particularly those expressed as linear combinations of outcome variables. Additionally, the course is designed to enhance students' proficiency in implementing these methods using statistical software and applying them to their dissertation research.

The course is structured according to data types, beginning with foundational concepts in quantitative social sciences and causal inference. Subsequently, we examine the design and analysis of randomized experiments featuring a single outcome and treatment indicator. In the second part, we discuss the role of covariates in experiments, along with the statistical properties of linear regression—a fundamental estimator to utilize covariate information. The third part delves into intricate experimental designs, such as block randomization and clustered experiments, and their parallels in observational studies premised on strong ignorability. It covers several techniques for adjusting the impacts of confounders, including matching, weighting, and balancing, culminating in methods to validate research designs through placebo tests and sensitivity analyses. The fourth part focuses on datasets with auxillary variables, such as the instrumental variable and the running variable in regression discontinuity designs. The fifth part is dedicated to panel data analysis, where each unit in the sample is observed for multiple periods. We will investigate common methods under different identification assumptions. Examples include the DID estimator, fixed effects models, synthetic control methods, trajectory balancing methods, and marginal structural models. The sixth part considers cases with multiple outcome variables, discussing mediation analysis and the complexities of multiple testing.

Throughout this course, we will adopt the design-based perspective, clearly differentiating between identification assumptions which guide the treatment assignment process and the structural restrictions that researchers apply to the interrelations among variables. A key emphasis will be placed on the preeminence of research design over the selection of specific statistical models. The underlying principle we'll reinforce is that causal identification stems primarily from the integrity of the research design, while statistical models serve as vital tools for estimation and inference.

Time and location

Classes: 9:30am - 10:45am on Tuesdays and Thursdays, Room 351 in Hamilton Hall.
There will no class on Feb. 13 (Well-being Day), Mar. 12 (Spring Break), Mar. 14 (Spring Break), or Mar. 28 (Well-being Day).
Instructor: Ye Wang (yewang@unc.edu).
Office hours: 1pm - 4pm on Wednesdays, Room 322 in Hamilton Hall.
Labs: 09:05am - 09:55am on Fridays, Room 0220 in Phillips Hall.
Teaching assistants: Matias Tarillo (mtarillo@unc.edu) and Tyler Ditmore (tditmore@unc.edu).
Office hours: 11am - 12:30pm on Tuesdays and Thursdays, Room 300 in Hamilton Hall.

Texts and software

The course will draw a lot from the following textbooks:

1. Angrist and Pischke (2008)
2. Imbens and Rubin (2015)

We may also refer to other textbooks and research papers for certain topics. More details can be found in the section on course outline. It is **not** required that students must read all the materials listed as references. But they are useful complements to the lecture notes.

We will working with R in this course, which is an open-source computing language that is very widely used in statistics. You can download it for free from www.r-project.org. You are also encouraged to use Rmarkdown for your homework.

Requirements

Students enrolling in this course are expected to have a solid understanding of probability theory, matrix algebra, and calculus, along with practical experience in scripting with R. It is highly recommended that participants have completed POLI-783 offered by the Department of Political Science or a comparable course.

The course grade will be composed of class participation (15%), six homework assignments (60%), and a final project (25%). Submissions for each assignment, including both the write-up and the R script, must be uploaded to Canvas before the specified deadline, typically prior to Thursday's class. Assignments generally involve three components: theoretical derivations, a simulation task, and a replication exercise. For the final project, students are required to conduct an extensive replication of a selected paper, applying methodologies learned throughout the course to reanalyze the core findings.

Course outline

Part I: Y and D

Lecture 0 (Jan. 11): Introduction to the course

Introduce the course.
Linear methods.
Identification assumptions vs. structural restrictions.
The design-based perspective.

Lecture 1 (Jan. 16): Basic concepts of empirical analysis

Estimand, estimator, and estimate.
Bias and consistency.
Variance, variance estimation, and efficiency.
Asymptotic distribution and statistical inference.
Coverage rate.

References: Ch2 of Friedman, Hastie, and Tibshirani (2001)

Lecture 2 (Jan. 18): The potential outcome framework

The Neyman-Rubin model.
The fundamental problem of causal inference: two solutions.
Random assignment.
Complete randomization vs. Bernoulli trial.
Horvitz-Thompson estimator vs. Hajek estimator.

References: Holland (1986), Samii (2016), Ch1 of Angrist and Pischke (2008), Ch1 and Ch2 of Imbens and Rubin (2015)

Lab 1 (Jan. 19): Running simulation in R and treatment assignment

Lecture 3 (Jan. 23): Statistical inference I

Sampling-based uncertainty vs. design-based uncertainty.
Neyman variance and its estimation.
From unbiasedness to consistency.
Large sample inference.

References: Ch3, Ch4 and Ch6 of Imbens and Rubin (2015), Abadie et al. (2020), Li and Ding (2017)

Lecture 4 (Jan. 25): Statistical inference II

Fisher's randomization test.
Bootstrap and jackknife.

References: Ch5 of Imbens and Rubin (2015), Young (2019), Ch 10 of Hansen (2016)

Lab 2 (Jan. 26): Estimation and inference in experiments I

Lecture 5 (Jan. 30): Regression analysis I

Review bivariate and multivariate regression.
Statistical properties of the OLS estimator.

References: Ch2 and Ch3 of Hansen (2016), Ch3 and Ch6 of Ding (2024)

Feb. 1 Assignment 1 due

Part II: Y, D, and X (covariates)

Lecture 6 (Feb. 1): Multivariate regression

The OLS estimator and causality.
The Frisch–Waugh–Lovell theorem.
Control for covariates under random assignment.
Lin's regression.

References: Samii and Aronow (2012), Lin (2013), Ch7 of Imbens and Rubin (2015)

Lab 3 (Feb. 2): Linear regression

Lecture 7 (Feb. 6): Heterogeneous treatment effects I

The moderator effect.
Interpret the interactive effect.
Caveats of interactions.
External validity.
Optimal assignment.

References: Egami and Hartman (2020), Kitagawa and Tetenov (2018)

Lecture 8 (Feb. 8): Heterogeneous treatment effects II

Nonparametric regression.
Kernels.
Estimating the moderator effect with kernel regression.

References: Hainmueller, Mummolo, and Xu (2019)

Lab 4 (Feb. 9): Estimation of the CATE

Part III: Y, D, and X (confounders)

Lecture 9 (Feb. 15): Complex experimental design

Block randomization.
Clustering experiments.
Clustered standard errors.

References: Ch9 of Imbens and Rubin (2015), Abadie et al. (2017)

Lab 5 (Feb. 16): Block randomization and cluster standard errors

Lecture 10 (Feb. 20): From experiments to observational studies

Strong ignorability.
The curse of dimensionality.
The central role of propensity score.

References: Ch12 and Ch13 of Imbens and Rubin (2015)

Feb. 22 Assignment 2 due

Lecture 11 (Feb. 22): Matching

Nearest-neighbor matching.
Propensity score matching.
Statistical inference of matching.

References: Abadie and Imbens (2006), Abadie and Imbens (2008), Iacus, King, and Porro (2012)

Lab 6 (Feb. 23): Matching

Lecture 12 (Feb. 27): Weighting and GLM I

Inverse probability of treatment weighting estimators.
Estimate the propensity scores.
Link function and logistic regression.

References: Ch20 Ding (2024)

Lecture 13 (Feb. 29): Weighting and GLM II

Properties of GLM.
Pros and cons of the IPW estimators.

References: Hirano, Imbens, and Ridder (2003)

Lab 7 (Feb. 30): GLM and weighting

Lecture 14 (Mar. 5): Balancing and regression

Entropy balancing.
Covariate balancing propensity scores.
Assumptions behind regression.

References: Hainmueller (2012), Imai and Ratkovic (2014), Aronow and Samii (2016)

Lecture 15 (Mar. 7): Advanced techniques

Selecting control variables.
Doubly robust estimators.
Machine learning in causal inference.

References: Ratkovic (2019)

Lab 8 (Mar. 8): Balance and doubly robust estimators

Mar. 14 Assignment 3 due

Lecture 16 (Mar. 19): Validate your research design

Placebo tests.
Sensitivity analyses.

References: Ch21 and Ch22 of Imbens and Rubin (2015), Cinelli and Hazlett (2020)

Part IV: Y, D, and Z

Lecture 17 (Mar. 21): Non-compliance I

Principal strata and the LATE.
Assumptions for identifying the LATE.
The Wald estimator.

References: Ch23 and Ch24 of Imbens and Rubin (2015)

Lab 9 (Mar. 22): Sensitivity analyses and non-compliance

Lecture 18 (Mar. 26): Non-compliance II

IV from the econometric perspective.
Estimation and inference in 2SLS.
Connections between the LATE and the 2SLS.

References: Angrist, Imbens, and Rubin (1996), Ch3 of Angrist and Pischke (2008)

Lecture 19 (Apr. 2): Non-compliance III

Weak IV.
Test of IV validity.
IV in practice.

References: Kitagawa (2015), Huber and Mellace (2015), Imbens and Newey (2009), Lee et al. (2022)

Apr. 4 Assignment 4 due

Lecture 20 (Apr. 4): Regression discontinuity design I

Identification in RDD.
Bandwidth selection.
Inference in RDD.

References: Lee and Lemieux (2010), McCrary (2008), Gelman and Imbens (2019), Calonico, Cattaneo, and Titiunik (2014), Calonico et al. (2019)

Lab 10 (Apr. 5): 2SLS and rdrobust I

Lecture 23 (Apr. 9): Regression discontinuity design II

Kink.
Fuzzy RDD.
Extrapolation.
RD in time.

References: Card et al. (2015), Keele and Titiunik (2015), Cattaneo et al. (2020)

Part V: Y, D, and T

Lecture 24 (Apr. 11): Panel data I

Strict exogeneity vs. sequential ignorability.
Additive fixed effects model.
The within estimator.

References: Blackwell (2013), Imai and Kim (2019), Ch3 of Hsiao (2014)

Lab 11 (Apr. 12): rdrobust II

Lecture 25 (Apr. 16): Panel data II

The DID estimator.
Staggered adoption and problems caused by HTE.
FEct.

References: Bertrand, Duflo, and Mullainathan (2004), Chaisemartin and D'Haultfœuille (2020), Strezhnev (2017), Liu, Wang, and Xu (2020)

Apr. 18 Assignment 5 due

Lecture 26 (Apr. 18): Panel data III

Dynamic treatment effects.
Synthetic control.
Balancing estimators.

References: Blackwell and Glynn (2018), Abadie, Diamond, and Hainmueller (2015), Hazlett and Xu (2018), Arkhangelsky et al. (2019)

Lab 12 (Apr. 19): FEct I

Lecture 27 (Apr. 23): Panel data IV

MSM.
Interactive fixed effects models.

References: Xu (2017), Athey et al. (2018)

Part VI: Ys

Lecture 30 (Apr. 25): Mediation analysis

Mediation analysis.
Multiple mediators.

References: Imai, Keele, and Tingley (2010), Acharya, Blackwell, and Sen (2016), Zhou and Yamamoto (2020)

Lab 13 (Apr. 26): PanelMatch, tjbal, and mediation analysis

Lecture 29 (Apr. 30): Multiple testing

Test the balance of covariates.
Multiple comparisons.
Review.

References: Hartman and Hidalgo (2018), Casey, Glennerster, and Miguel (2012), Anderson (2008), Viviano, Wuthrich, and Niehaus (2021)

May 2 Assignment 6 due

May 9 Final project due

References:

Abadie, Alberto, Susan Athey, Guido W Imbens, and Jeffrey Wooldridge. 2017. "When Should You Adjust Standard Errors for Clustering?" National Bureau of Economic Research.

Abadie, Alberto, Susan Athey, Guido W Imbens, and Jeffrey M Wooldridge. 2020. "Sampling-Based Versus Design-Based Uncertainty in Regression Analysis." *Econometrica* 88 (1): 265–96.

Abadie, Alberto, Alexis Diamond, and Jens Hainmueller. 2015. "Comparative Politics and the Synthetic Control Method." *American Journal of Political Science* 59 (2): 495–510.

Abadie, Alberto, and Guido W Imbens. 2006. "Large Sample Properties of Matching Estimators for Average Treatment Effects." *Econometrica* 74 (1): 235–67.

———. 2008. "On the Failure of the Bootstrap for Matching Estimators." *Econometrica* 76 (6): 1537–57.

Acharya, Avidit, Matthew Blackwell, and Maya Sen. 2016. "Explaining Causal Findings Without Bias: Detecting and Assessing Direct Effects." *American Political Science Review* 110 (3): 512–29.

Anderson, Michael L. 2008. "Multiple Inference and Gender Differences in the Effects of Early Intervention: A Reevaluation of the Abecedarian, Perry Preschool, and Early Training Projects." *Journal of the American Statistical Association* 103 (484): 1481–95.

Angrist, Joshua D, Guido W Imbens, and Donald B Rubin. 1996. "Identification of Causal Effects Using Instrumental Variables." *Journal of the American Statistical Association* 91 (434): 444–55.

Angrist, Joshua D, and Jörn-Steffen Pischke. 2008. *Mostly Harmless Econometrics: An Empiricist's Companion*. Princeton university press.

Arkhangelsky, Dmitry, Susan Athey, David A Hirshberg, Guido W Imbens, and Stefan Wager. 2019. "Synthetic Difference in Differences." National Bureau of Economic Research.

Aronow, Peter M, and Cyrus Samii. 2016. "Does Regression Produce Representative Estimates of Causal Effects?" *American Journal of Political Science* 60 (1): 250–67.

Athey, Susan, Mohsen Bayati, Nikolay Doudchenko, Guido Imbens, and Khashayar Khosravi. 2018. "Matrix Completion Methods for Causal Panel Data Models." National Bureau of Economic Research.

Bertrand, Marianne, Esther Duflo, and Sendhil Mullainathan. 2004. "How Much Should We Trust Differences-in-Differences Estimates?" *The Quarterly Journal of Economics* 119 (1): 249–75.

Blackwell, Matthew. 2013. "A Framework for Dynamic Causal Inference in Political Science." *American Journal of Political Science* 57 (2): 504–20.

Blackwell, Matthew, and Adam N Glynn. 2018. "How to Make Causal Inferences with Time-Series Cross-Sectional Data Under Selection on Observables." *American Political Science Review* 112 (4): 1067–82.

Calonico, Sebastian, Matias D Cattaneo, Max H Farrell, and Rocio Titiunik. 2019. "Regression Discontinuity Designs Using Covariates." *Review of Economics and Statistics* 101 (3): 442–51.

Calonico, Sebastian, Matias D Cattaneo, and Rocio Titiunik. 2014. "Robust Nonparametric Confidence Intervals for Regression-Discontinuity Designs." *Econometrica* 82 (6): 2295–2326.

Card, David, David S Lee, Zhuan Pei, and Andrea Weber. 2015. "Inference on Causal Effects in a Generalized Regression Kink Design." *Econometrica* 83 (6): 2453–83.

Casey, Katherine, Rachel Glennerster, and Edward Miguel. 2012. "Reshaping Institutions: Evidence on Aid Impacts Using a Preanalysis Plan." *The Quarterly Journal of Economics* 127 (4): 1755–1812.

Cattaneo, Matias D, Luke Keele, Rocio Titiunik, and Gonzalo Vazquez-Bare. 2020. "Extrapolating Treatment Effects in Multi-Cutoff Regression Discontinuity Designs." *Journal of the American Statistical Association*, 1–12.

Chaisemartin, Clément de, and Xavier D'Haultfœuille. 2020. "Two-Way Fixed Effects Estimators with Heterogeneous Treatment Effects." *American Economic Review*.

Cinelli, Carlos, and Chad Hazlett. 2020. "Making Sense of Sensitivity: Extending Omitted Variable Bias." *Journal of the Royal Statistical Society: Series B (Statistical Methodology)* 82 (1): 39–67.

Ding, Peng. 2024. "Linear Model and Extensions." *arXiv Preprint arXiv:2401.00649*.

Egami, Naoki, and Erin Hartman. 2020. "Elements of External Validity: Framework, Design, and Analysis."

Friedman, Jerome, Trevor Hastie, and Robert Tibshirani. 2001. *The Elements of Statistical Learning*. Vol. 1. 10. Springer series in statistics New York.

Gelman, Andrew, and Guido Imbens. 2019. "Why High-Order Polynomials Should Not Be Used in Regression Discontinuity Designs." *Journal of Business & Economic Statistics* 37 (3): 447–56.

Hainmueller, Jens. 2012. "Entropy Balancing for Causal Effects: A Multivariate Reweighting Method to

Produce Balanced Samples in Observational Studies." *Political Analysis*, 25–46.

Hainmueller, Jens, Jonathan Mummolo, and Yiqing Xu. 2019. "How Much Should We Trust Estimates from Multiplicative Interaction Models? Simple Tools to Improve Empirical Practice." *Political Analysis* 27 (2): 163–92.

Hansen, Bruce. 2016. "Econometrics." *A Textbook Draft Available Online at Www. Ssc. Wisc. Edu/~ Bhansen/Econometrics/Econometrics. Pdf.*

Hartman, Erin, and F Daniel Hidalgo. 2018. "An Equivalence Approach to Balance and Placebo Tests." *American Journal of Political Science* 62 (4): 1000–1013.

Hazlett, Chad, and Yiqing Xu. 2018. "Trajectory Balancing: A General Reweighting Approach to Causal Inference with Time-Series Cross-Sectional Data." *Available at SSRN 3214231*.

Hirano, Keisuke, Guido W Imbens, and Geert Ridder. 2003. "Efficient Estimation of Average Treatment Effects Using the Estimated Propensity Score." *Econometrica* 71 (4): 1161–89.

Holland, Paul W. 1986. "Statistics and Causal Inference." *Journal of the American Statistical Association* 81 (396): 945–60.

Hsiao, Cheng. 2014. *Analysis of Panel Data*. 54. Cambridge university press.

Huber, Martin, and Giovanni Mellace. 2015. "Testing Instrument Validity for LATE Identification Based on Inequality Moment Constraints." *Review of Economics and Statistics* 97 (2): 398–411.

Iacus, Stefano M, Gary King, and Giuseppe Porro. 2012. "Causal Inference Without Balance Checking: Coarsened Exact Matching." *Political Analysis*, 1–24.

Imai, Kosuke, Luke Keele, and Dustin Tingley. 2010. "A General Approach to Causal Mediation Analysis." *Psychological Methods* 15 (4): 309.

Imai, Kosuke, and In Song Kim. 2019. "When Should We Use Unit Fixed Effects Regression Models for Causal Inference with Longitudinal Data?" *American Journal of Political Science* 63 (2): 467–90.

Imai, Kosuke, and Marc Ratkovic. 2014. "Covariate Balancing Propensity Score." *Journal of the Royal Statistical Society: Series B: Statistical Methodology*, 243–63.

Imbens, Guido W, and Whitney K Newey. 2009. "Identification and Estimation of Triangular Simultaneous Equations Models Without Additivity." *Econometrica* 77 (5): 1481–1512.

Imbens, Guido W, and Donald B Rubin. 2015. *Causal Inference in Statistics, Social, and Biomedical Sciences*. Cambridge University Press.

Keele, Luke J, and Rocio Titiunik. 2015. "Geographic Boundaries as Regression Discontinuities." *Political Analysis* 23 (1): 127–55.

Kitagawa, Toru. 2015. "A Test for Instrument Validity." *Econometrica* 83 (5): 2043–63.

Kitagawa, Toru, and Aleksey Tetenov. 2018. "Who Should Be Treated? Empirical Welfare Maximization Methods for Treatment Choice." *Econometrica* 86 (2): 591–616.

Lee, David S, and Thomas Lemieux. 2010. "Regression Discontinuity Designs in Economics." *Journal of Economic Literature* 48 (2): 281–355.

Lee, David S, Justin McCrary, Marcelo J Moreira, and Jack Porter. 2022. "Valid t-Ratio Inference for IV." *American Economic Review* 112 (10): 3260–90.

Li, Xinran, and Peng Ding. 2017. "General Forms of Finite Population Central Limit Theorems with Applications to Causal Inference." *Journal of the American Statistical Association* 112 (520): 1759–69.

Lin, Winston. 2013. "Agnostic Notes on Regression Adjustments to Experimental Data: Reexamining Freedman's Critique." *The Annals of Applied Statistics* 7 (1): 295–318.

Liu, Licheng, Ye Wang, and Yiqing Xu. 2020. "A Practical Guide to Counterfactual Estimators for Causal Inference with Time-Series Cross-Sectional Data." *Available at SSRN 3555463*.

McCrary, Justin. 2008. "Manipulation of the Running Variable in the Regression Discontinuity Design: A Density Test." *Journal of Econometrics* 142 (2): 698–714.

Ratkovic, Marc. 2019. "Rehabilitating the Regression: Honest and Valid Causal Inference Through Machine Learning."

Samii, Cyrus. 2016. "Causal Empiricism in Quantitative Research." *The Journal of Politics* 78 (3): 941–55.

Samii, Cyrus, and Peter M Aronow. 2012. "On Equivalencies Between Design-Based and Regression-Based Variance Estimators for Randomized Experiments." *Statistics & Probability Letters* 82 (2): 365–70.

Strezhnev, Anton. 2017. "Generalized Difference-in-Differences Estimands and Synthetic Controls." *Unpublished Manuscript*.

Viviano, Davide, Kaspar Wuthrich, and Paul Niehaus. 2021. "(When) Should You Adjust Inferences for

Multiple Hypothesis Testing?” *arXiv Preprint arXiv:2104.13367*.
Xu, Yiqing. 2017. “Generalized Synthetic Control Method: Causal Inference with Interactive Fixed Effects Models.” *Political Analysis* 25 (1): 57–76.
Young, Alwyn. 2019. “Channeling Fisher: Randomization Tests and the Statistical Insignificance of Seemingly Significant Experimental Results.” *The Quarterly Journal of Economics* 134 (2): 557–98.
Zhou, Xiang, and Teppei Yamamoto. 2020. “Tracing Causal Paths from Experimental and Observational Data.”