Instrumental Variable III

Ye Wang
University of North Carolina at Chapel Hill

Linear Methods in Causal Inference
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Review

- The previous lecture focuses on the economic tradition of using IVs.
- It is seen as an exogenous factor that may affect an agent's rational choice in the triangular system.
- Common approaches to use an IV include 2SLS and control function.
- There exists a deep connection between an IV and non-compliance conceptually.
- Results from 2SLS can be identical to that from the Wald estimator under certain conditions.

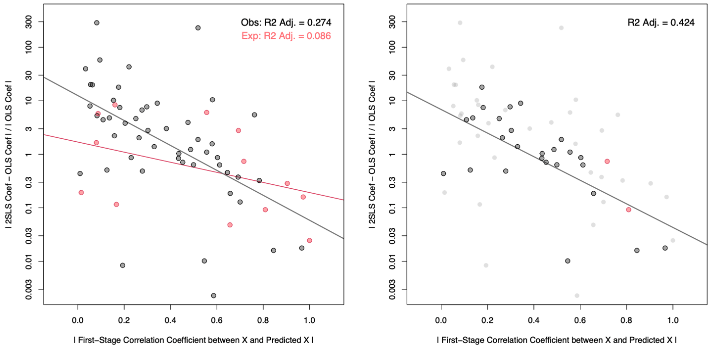
Testing assumptions in IV estimation

- We have seen that the LATE framework and the triangular system rely on similar assumptions:
  1. random assignment of the instrument.
  2. exclusion restriction.
  3. first stage.
  4. monotonicity.
- It is easier to ensure that all these assumptions are satisfied in experiments.
- In observational studies, these assumptions are less plausible.
- It explains why IV is becoming less popular in observational studies.
- Testing their validity is necessary.

Weak instruments

- Let's start from an assumption that seems easy to be satisfied: first stage.
- Remember that $\hat{\tau} = \frac{\hat{\xi}}{\hat{\delta}}$.
- If Z_i and D_i are only weakly correlated, $\hat{\delta}$ will be close to zero and the finite sample bias of $\hat{\tau}$ will be huge.
- Staiger and Stock (1997): If $\delta = \frac{c}{\sqrt{N}}$, then $\hat{\tau} \to \tau + \eta$, where η is the ratio between two normally distributed variables.
- The bias may even larger than that from OLS.
- Lal et al. (2023) show that this is a common problem in political science.
- The difference between 2SLS and OLS is larger when the IV is weaker.

Weak instruments



(a) Full Sample (b) Subsample with Significant OLS Results

Weak instruments

- A well-known example is Angrist and Krueger (1991).
- They instrument students' schooling years with their season of birth.
- But the correlation between the two variables is very weak.
- Bound, Jaeger, and Baker (1995) show that replacing the actual season of birth with randomly assigned ones does not change the estimate.

Weak instruments

- The confidence intervals of $\hat{\tau}$ will not have the correct coverage when the IV is weak.
- A traditional rule of thumb for a strong IV is that $F = \frac{\hat{\delta}^2}{\widehat{V}(\hat{\delta})}$ is larger than 10 (Staiger and Stock 1997).
- This is not the overall F-statistic from the first stage regression.
- If there are multiple instruments, we should test the joint null hypothesis that all these coefficients are zero.
- Yet Lee et al. (2022) show that this is not sufficient.
- If we impose no restrictions on the DGP, F must be larger than 104.7 to ensure the correct coverage.
- Or we use the critical value of 3.43.

Weak instruments

- Intuitively, the distribution of $\hat{\tau}$ deviates from normality significantly if $\hat{\delta}$ is not distinguishable from zero.
- The degree of deviation hinges on the value of F.
- There are several solutions to this problem.
- Anderson and Rubin (1949) propose to test the null hypothesis $\xi = \xi_0 = \delta\tau_0$ instead of $\tau = \tau_0$.
- We can see that $\hat{\xi} - \hat{\delta}\tau_0 \rightarrow \mathcal{N}(0, \Sigma)$ under the null, where

$$\Sigma = Var[\hat{\xi}] - 2\tau_0 Cov[\hat{\delta}, \hat{\xi}] + \tau_0^2 Var[\hat{\delta}].$$

- Such a test is valid even when $\hat{\delta}$ is zero.

Weak instruments

- The Anderson-Rubin (AR) test avoids the issue of taking the ratio but the result is hard to interpret.
- Lee et al. (2022) note the following relationship

$$t^2 = \frac{t_{AR}^2}{1 - 2\rho \frac{t_{AR}}{F} + \frac{t_{AR}^2}{F^2}},$$

  where ρ is the correlation between ν_i and ε_i.
- The relative performance of the t-test to the AR test is decided by the value of F.
- They hence suggest that we should combine the tests from the first stage and the second stage.
- For each value of F, they provide the corresponding critical value in the second stage.
- Or, we can impose more restrictions on the DGP, such as ρ.

Weak instruments: application

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## The 2SLS estimate is 2.987609
## The first-stage F-statistic value is: 197.268
## The p-value from the Anderson-Rubin test is: 2.220446e-1
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Test exclusion restriction

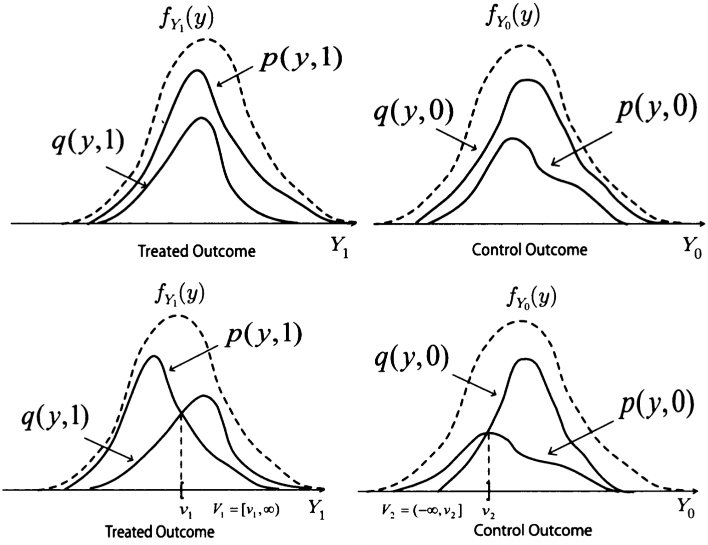
- Kitagawa (2015) and Huber and Mellace (2015) suggest that we may test exclusion restriction and monotonicity altogether.
- These two assumptions imply that

$$
\begin{aligned}
P(y, D=1|Z=1) &> P(y, D=1|Z=0) \\
P(y, D=0|Z=1) &< P(y, D=0|Z=0)
\end{aligned}
$$

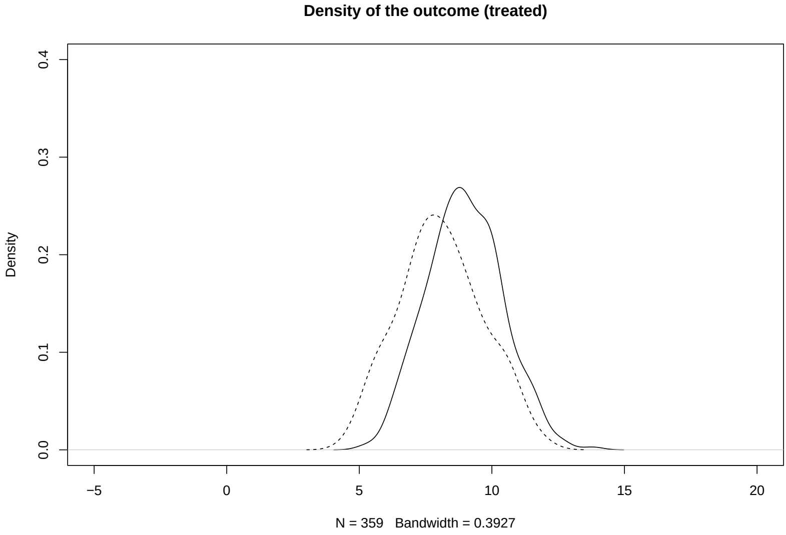
  for all y.
- We can test these two inequalities with data.
- To obtain critical values, we need to construct a variance-weighted Kolmogorov-Smirnov test statistic and apply bootstrap to it.
- But visualization alone can be helpful.

Test exclusion restriction

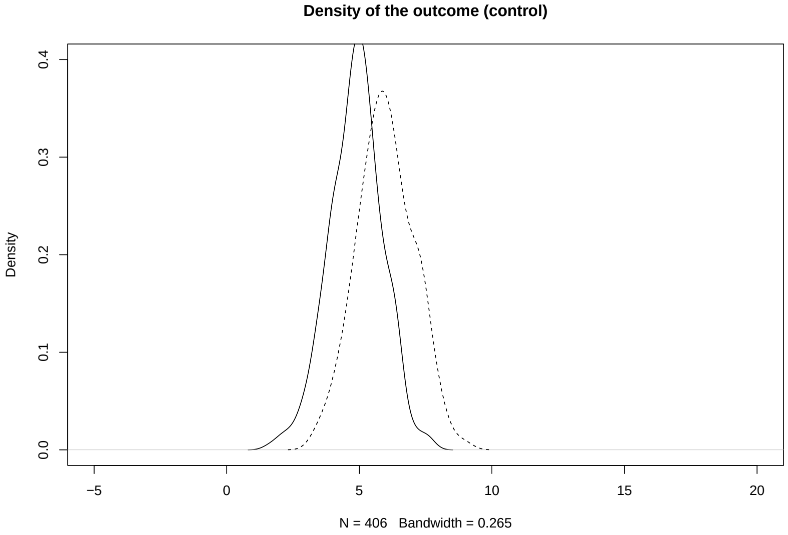
- We want to see the plots on the top rather than the ones on the bottom:



Test exclusion restriction: application



Test exclusion restriction: application



Exclusion restriction in practice

- It is common that exclusion restriction is violated in observational studies.
- Two popular instrumental variables in practice are rainfall and distance to the location where an event happened.
- Sarsons (2015) demonstrates that in Indian areas with dams, rainfall still affects conflicts even though it is not correlated with income.
- Zhao (2023) shows that using distance to any German city as the instrument can replicate the results in Becker and Woessmann (2009).
- The problem is that excluding all the other channels is more difficult than we thought.
- Glaeser et al. (2004) suggest that settlers' mortality may also influence the level of human capital in former colonies.

Generalizing the LATE

- It is reasonable to assume that the principal strata are determined by observable attributes of units.
- Abadie (2003) proposes a statistic (Abadie's κ):

$$\kappa_i = 1 - \frac{D_i(1 - Z_i)}{P(Z_i = 0|\mathbf{X}_i)} - \frac{(1 - D_i)Z_i}{P(Z_i = 1|\mathbf{X}_i)}.$$

- He shows that for any function $g(Y_i, D_i, \mathbf{X}_i)$,

$$\begin{aligned} & E[g(Y_i, D_i, \mathbf{X}_i)|D_i(1) > D_i(0)] \\ = & \frac{1}{P(D_i(1) > D_i(0))} E[\kappa_i g(Y_i, D_i, \mathbf{X}_i)]. \end{aligned}$$

- It means that we can estimate the average of any statistic over compliers by calculating its average weighted by κ_i.

Compliance score

- Under the same assumption, we can explicitly model the relationship between principal strata and covariates.
- We define $q_{ic} = q_c(\mathbf{X}_i)$ as the compliance score (Aronow and Carnegie 2013).
- We need to impose assumptions on the form of $q_c(\mathbf{X}_i)$.
- To estimate the ATE, we weight each observation with $\frac{1}{q_{ic}}$ and apply the Wald estimator.
- The intuition resembles the IPW estimator.

Compliance score

- Let's assume that

$$q_a(\mathbf{X}_i) + q_c(\mathbf{X}_i) = h(\mathbf{X}_i\beta_{a,c}),$$
$$\frac{q_a(\mathbf{X}_i)}{q_a(\mathbf{X}_i) + q_c(\mathbf{X}_i)} = h(\mathbf{X}_i\beta_{a|(a,c)}),$$

- We observe (D_i, Z_i) in the data with the likelihood

$$\prod_{i=1}^{N}(h(\mathbf{X}_i\beta_{a,c})(1 - h(\mathbf{X}_i\beta_{a|(a,c)}))Z_i + h(\mathbf{X}_i\beta_{a,c})h(\mathbf{X}_i\beta_{a|(a,c)}))^{D_i} *$$
$$(1 - h(\mathbf{X}_i\beta_{a,c})(1 - h(\mathbf{X}_i\beta_{a|(a,c)}))Z_i - h(\mathbf{X}_i\beta_{a,c})h(\mathbf{X}_i\beta_{a|(a,c)}))^{(1-D_i)}$$

- We estimate $\beta_{a,c}$ and $\beta_{a|(a,c)}$ via MLE and obtain the compliance score.

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