Quant II

TSCS Data Estimation

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3/25/2020

Outline

- What is unique about TSCS data?
- Inference without unobservable confounders
- Inference with unobservable confounders
  - Model-based inference
  - Trajectory-based inference

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- In the cross-sectional setting, we need ignorability: $D_i \perp Y_i(D_i)|X_i$, $0 < P(D_i = 1|X_i) < 1$.
- Now we can relax the assumption along two possible directions (based on your belief!):
  - Sequential ignorability: $D_{it} \perp Y_{it}(D_{it}, \ldots, D_{i1})|Y_{i,t-1}, D_{i,t-1}, X_{it}$.
  - "Fixed effects:" $D_{it} \perp Y_{it}(D_{it})|X_{it}, \alpha_i, \xi_t$

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  Make parametric assumptions

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  The lagged depedent variable does solve a lot of problems, as the time trend is usually the most important confounder.

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  How to estimate the effect of your treatment history?

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- We can obtain valid estimate for the history's aggregated effect after controlling for confounders.

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  Omitted variable bias for $D_{i,t-1}$.

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- The former models Y (response surface) and the latter models D (treatment assignment)

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  - Transform Y_{it} into $\tilde{Y}_{it} = Y_{it} - \hat{\gamma}_0 D_{it}$
  - Regress $\tilde{Y}_{it}$ on $(Y_{i,t-2}, D_{i,t-1}, D_{i,t-2})$ and get the coefficient of $D_{i,t-1}$, $\hat{\gamma}_1$.

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  - Regress $\tilde{Y}_{it}$ on $(Y_{i,t-2}, D_{i,t-1}, D_{i,t-2})$ and get the coefficient of $D_{i,t-1}$, $\hat{\gamma}_1$.
- MSMs with IPTWs: estimate the propensity score for each X_{it}, and use IPTW for ATT (Jim Robins's g-formula).

$$W_{it} = \prod_{t=1}^{t} Pr[D_{it} | D_{i,t-1}, Y_{i,t-1}]$$

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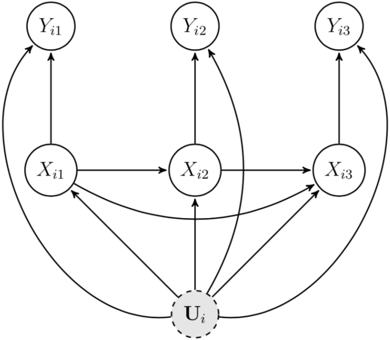

- No feedback, no carryover, no between-unit interference

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  Wang (2020): two-way FE models lead to meaningless estimates

The cost of allowing for unobservable confounders

- When confounders are unobservable, we cannot estimate the propensity score any more.
- Response surface adjustment is the only possible approach.
- Either we write down a DGP for the response surface or we impose assumptions on its “trajectory”

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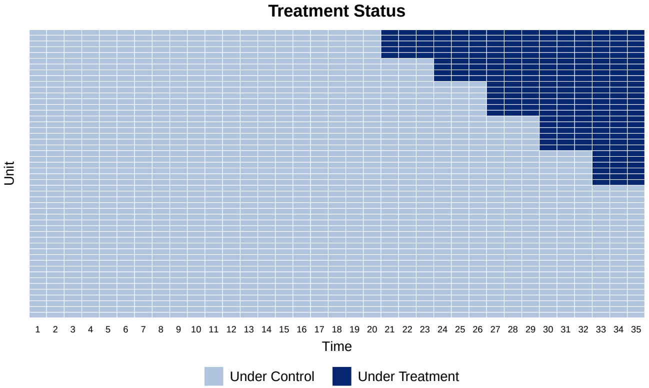

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- We can make the error structure more complicated by replacing $\alpha_i + \zeta_t$ with $\lambda_i \mathbf{f}_t$, where $\lambda_i = (\lambda_1, \lambda_2, \ldots, \lambda_r)$ and $\mathbf{f}_t = (f_1, f_2, \ldots, f_r)$ (interactive FE).
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  How many parameters we need to estimate?
- From a different perspective, we are approximating the matrix $\mathbf{Y} = \{Y_{it}\}$ with the product of two low-dimension matrices: $\mathbf{Y} = \mathbf{L} + \varepsilon$, where $\mathbf{L} = \Lambda\mathbf{F}$ (matrix completion or MC).

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  IFE and MC are equivalent when MC directly penalizes the matrix dimension r (hard-impute) rather than the magnitude of eigen values (soft-impute).

Semi-parametric models: estimation

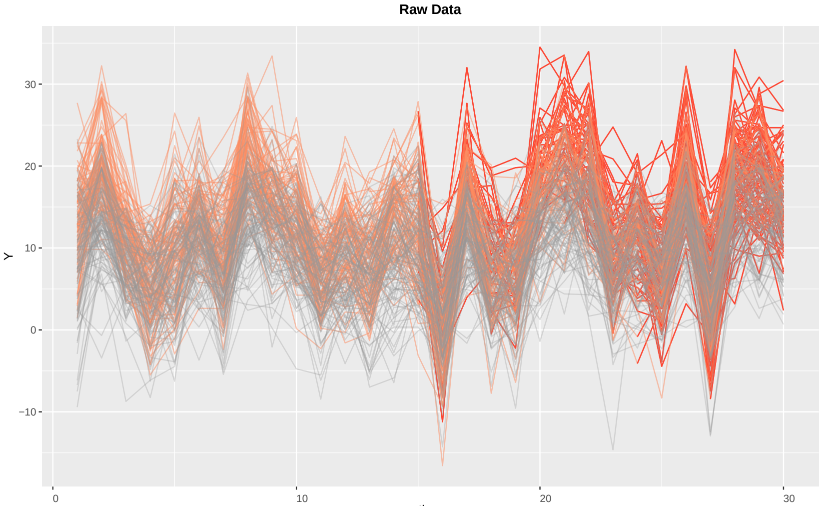
- IFE relies on the singular value decomposition (SVD).
- We start from initial parameters, estimate β via OLS, and obtain λ_i and $\mathbf{f}_t$ via SVD of the residuals.
- r is selected by cross-validation.
- MC (soft-impute) uses the direct sum decomposition: the residual at each round can be decomposed into two orthogonal parts.
- MC (soft-impute) is biased in finite samples.
- Both work well in large samples (relative performance depends on the strength of factors).
- We can add other parts to the model such as the lagged dependent variable or ensemble methods (Athey et al., 2019)

Semi-parametric models: tools

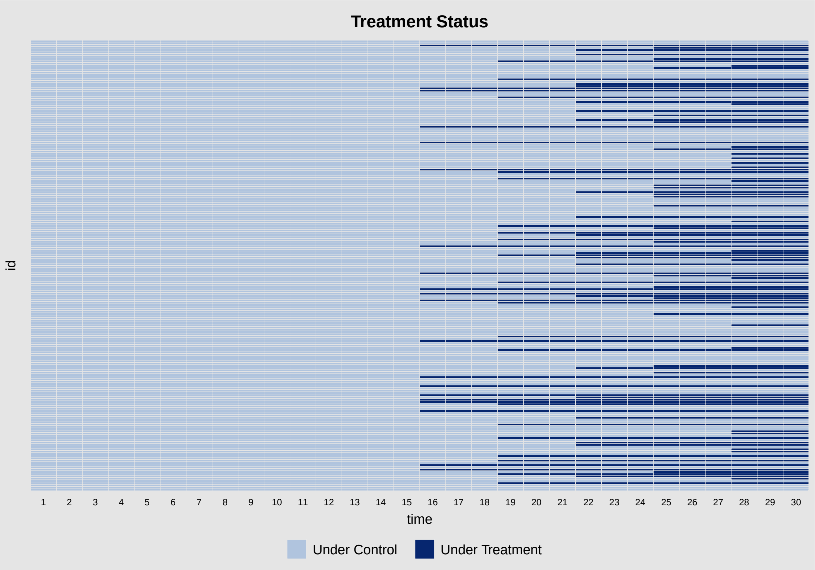
- A series of packages provided by Yiqing Xu at Stanford and his collaborators
- panelView for displaying the basic patterns
- fastplm for estimating the two-way FE models fast
- gsynth for estimating the interactive FE models
- fect to rule them all

Example

```
## Registered S3 method overwritten by 'GGally':
##   method from
##   +.gg   ggplot2
```



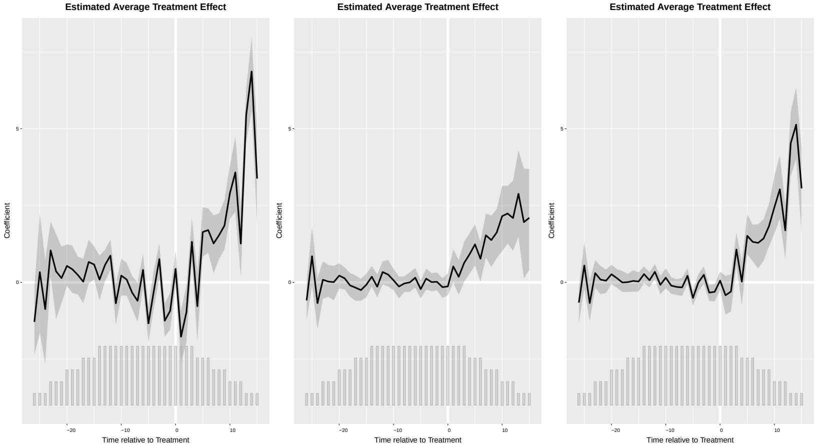
Example


Treatment Status
id
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30
time
Under Control
Under Treatment

Example

```
##
##     Coef Std. Error t value Pr(>|t|) CI_lower CI_upper
## D  0.249      0.163   1.530        0   -0.070    0.569
## X1 0.999      0.041  24.432        0    0.918    1.079
## X2 2.941      0.042  69.729        0    2.858    3.024
##
## ---
##
## Residual standard error: 3.171 on 5768 degrees of freedo
## Multiple R-squared(full model): 0.800    Adjusted R-squa
## Multiple R-squared(proj model): 0.496    Adjusted R-squa
## Wald F-statistic: 1856.764 on 3 and 5768 DF, p-value: <
##
## ---
##
## Variance Type:  Robust
```

Example



Estimated Average Treatment Effect
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Coefficient
5
0
−20
−10
0
10
Time relative to Treatment

Assumptions behind the semi-parametric models

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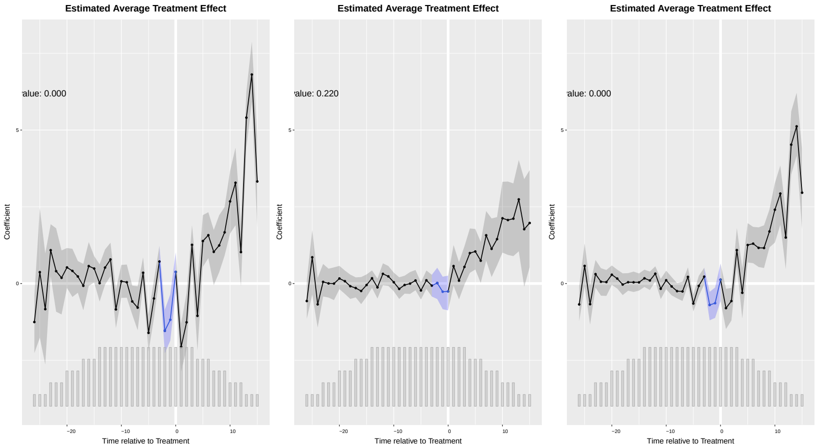
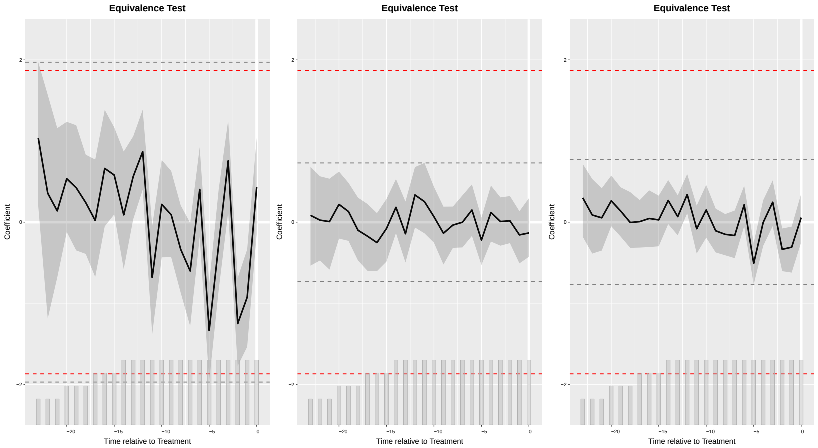
- Liu, Wang, and Xu (2019): dynamic treatment effects, equivalence test and placebo test.

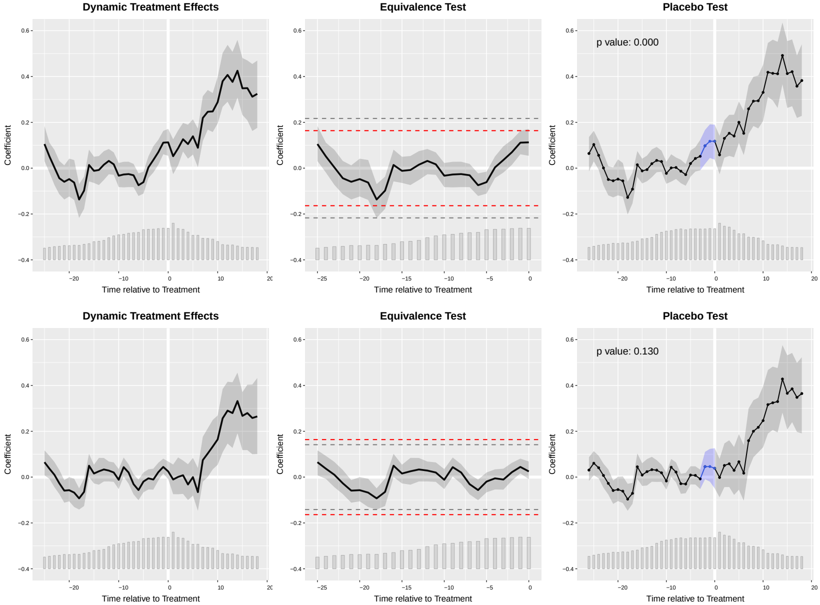
Example


Estimated Average Treatment Effect
value: 0.000
Coefficient
Time relative to Treatment
Estimated Average Treatment Effect
value: 0.220
Coefficient
Time relative to Treatment
Estimated Average Treatment Effect
value: 0.000
Coefficient
Time relative to Treatment

Example


Equivalence Test
Equivalence Test
Equivalence Test
Coefficient
Coefficient
Coefficient
2
0
−2
−20
−15
−10
−5
0
Time relative to Treatment
Time relative to Treatment
Time relative to Treatment

Example: Tomz et al. (2007)



Trajectory balancing

- One common assumption: conditional parallel trends

$$E[Y_{it}(0) - Y_{i,t-1}(0) | D_{it} = 1, X_{it}] = E[Y_{it}(0) - Y_{i,t-1}(0) | D_{it} = 0, X_{it}]$$

- Then $E[Y_{it}(1) - Y_{i,t-1}(0) | D_{it} = 1, X_{it}] = E[Y_{it}(1) | D_{it} = 1, X_{it}] - E[Y_{i,t-1}(0) | D_{it} = 1, X_{it}] - E[Y_{it}(0) | D_{it} = 0, X_{it}] + E[Y_{i,t-1}(0) | D_{it} = 0, X_{it}]$
- Synthetic control, Semi-parametric DID (Abadie, 2005), kernel balancing (Hazlett and Xu; 2017), panel matching (Imai, Kim, and Wang; 2018)
- One can use weighting to adjust the difference in trends among groups.
- Remember that this is not IPW and we can only get ATT not ATE.

A new hope

- Arkhangelsky and Imbens (2019)
- Even if we allow for the existence of unobservable confounders, it may still be possible to estimate the propensity scores.
- We need stronger assumptions on the assignment mechanism.
- For example, when the true propensity score has the form:

$$E\left[D_{it}|\alpha_i\right] = \frac{exp(\alpha_i + \xi_t)}{1 + exp(\alpha_i + \xi_t)}$$

  We have $\mathbf{D}_i^T \perp \mathbf{Y}_{jt}(\mathbf{D}^T)|\frac{1}{T}\sum_{t=1}^{T} D_{it}$.
- They show that there exists a doubly-robust estimator under these assumptions.