Panel Data Analysis IV

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Linear Methods in Causal Inference
POLI784

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- They allow for the interaction between unit and period fixed effects hence are more flexible.
- They can be estimated by eigenvalue decomposition.
- If there is only one treated unit, we can direct weight untreated units to eliminate the influence of factors.
- This synthetic control method is designed for case study and does not have real confidence intervals.
- Otherwise, it is better to estimate the factors and factor loadings first, and use them to predict the counterfactual outcomes.

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- We still assume no anticipation or cross-unit interference.
- If treatment assignments over periods are dependent, then past treatment status become confounders.
- We must control their influence under either strict exogeneity or sequential ignorability.

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- We can still estimate $\tau_{t,ATT}$ with

$$\begin{aligned}\hat{\tau}_t =& \frac{1}{|\mathcal{T}|}\sum_{i\in\mathcal{T}}\left(Y_{it} - \hat{Y}_{it}\left(\mathbf{0}^{1:t}\right)\right).\\ =& \frac{1}{|\mathcal{T}|}\sum_{i\in\mathcal{T}}\left(Y_{it}\left(\mathbf{0}^{1:T_0}, \mathbf{1}^{(T_0+1):t}\right) - \hat{Y}_{it}\left(\mathbf{0}^{1:t}\right)\right).\end{aligned}$$

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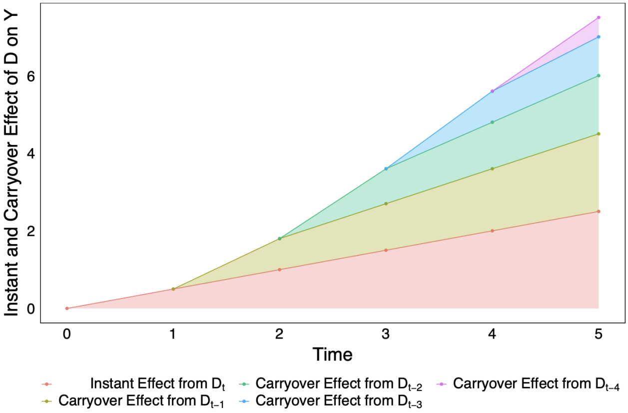
- We can show that the estimate converges to the cumulative effect of the treatment assignment history: $\frac{1}{|\mathcal{T}|}\sum_{i\in\mathcal{T}}\sum_{s=T_0+1}^{t}\tau_{is}$.



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- Any Y_{it} for $t > T_0$ is a mediator rather a confounder.
- All we need to do is to balance $\mathbf{Y}_i^{1:T_0}$ across the two groups.
- As suggested by the method of synthetic control, it accounts for the influence of both the outcome history and unobservable confounders.
- α_i can be learned from $\mathbf{Y}_i^{1:T_0}$, hence controlling for $\mathbf{Y}_i^{1:T_0}$ controls for α_i.

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- Next, we estimate the treatment effect on the treated in each period

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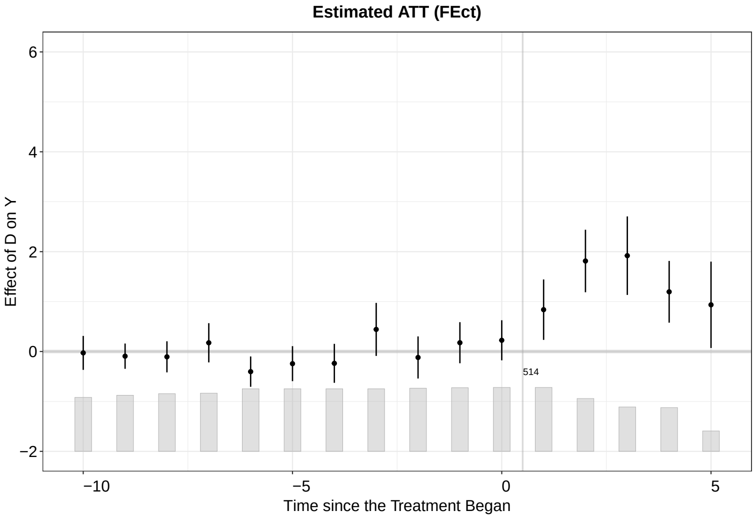
- They show the consistency of this estimator when the potential outcomes are linear in $f\left(\mathbf{Y}_i^{1:T_0}\right)$.
- Kim, Imai, and Wang (2019) argue that we can do the same via matching.

Application

- Let's revisit Hainmueller and Hangartner (2019).



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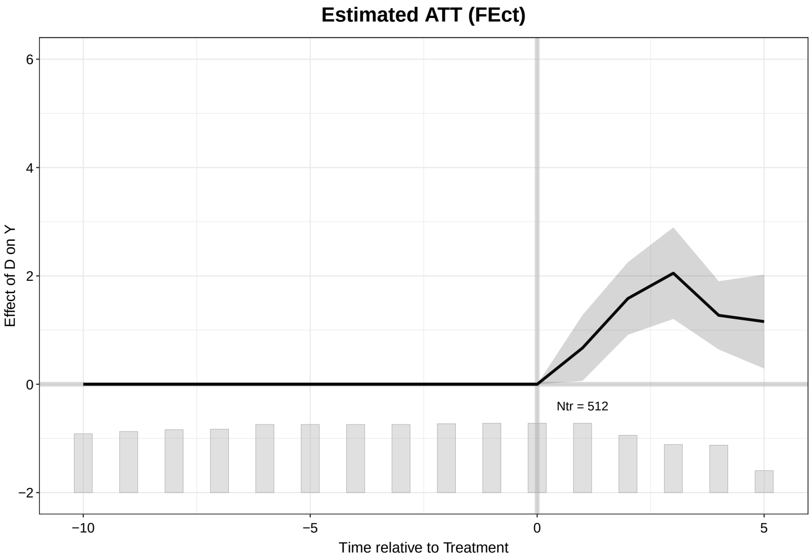

Carryover under treatment reversal

- In this case, our outcome model becomes

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Y_{it} =& \tau_{it} D_{it} + \sum_{s=1}^{t-1} \tau_{is} D_{is} + f_t\left(\mathbf{X}_{it}\right) + h_t\left(\mathbf{U}_i\right) + \varepsilon_{it} \\
=& \tau_{it} D_{it} + v_t\left(\mathbf{D}_i^{1:(t-1)}, \mathbf{X}_{it}, \mathbf{U}_i\right) + f_t\left(\mathbf{X}_{it}\right) + h_t\left(\mathbf{U}_i\right) + \varepsilon_{it},
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- We cannot learn the unobservable confounders from untreated observations.

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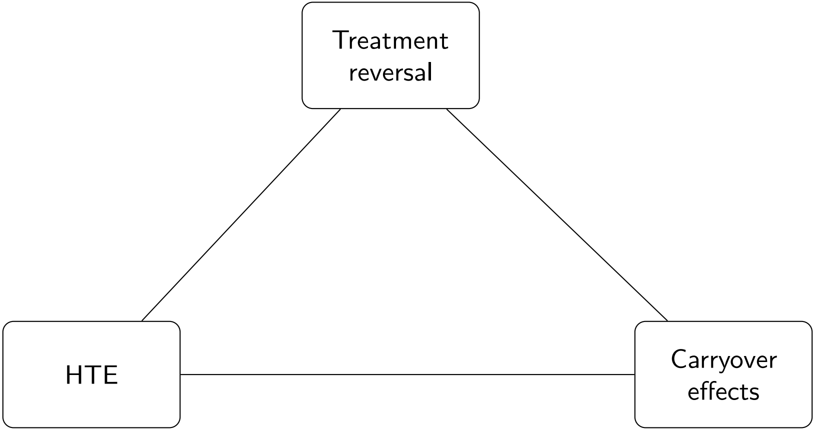
- Wang and Xu (2024) argue that there exists a trilemma for methods under strict exogeneity:



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- Another solution is drop periods after treatment reversal.

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- Another solution is drop periods after treatment reversal.
- But it changes the estimand and leads to efficiency loss.

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- Think about the "target trial" or "ideal experiment" before starting your analysis.
- Justify the structural restrictions with various methods or models.
- Validate the identification assumptions via placebo tests or sensitivity analysis.
- Do not seek for causality all the time.
- Be honest and transparent to yourself and your readers.

References I

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