Evaluate Your Research Design

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Linear Methods in Causal Inference
POLI784

Review

- Under strong ignorability, we can combine various methods to construct doubly robust estimators.
- The estimates they generate will be credible when one of the nuisance parameters is correctly estimated.
- Examples include the AIPW estimator and the bias-correction matching estimator.
- If we have a large number of covariates and are uncertain about the set of confounders, machine learning algorithms can be helpful.
- By combining penalization and cross-validation, they can estimate the nuisance parameters accurately without knowing what covariates to control for.
- To remove regularization bias from these algorithms, we need estimators which satisfy Neyman orthogonality and apply cross-fitting.

Bad controls

- The validity of unconfoundedness is built upon the correct choice of confounders.
- In theory, this should be decided by our substantive knowledge.
- Machine learning can assist us in this process.
- There are also some principles we should follow.
- We should not include any variable that can be affected by the treatment (post-treatment variable).
- E.g., controlling today's GDP per capita when studying the impact of a historic event on public opinion.
- A post-treatment variable plays the role of a mediator.
- It may attenuate the effect generated by the treatment and causes bias.

Post-treatment bias

- A post-treatment variable $S_i \in \{0, 1\}$ is a function of D_i:

$$S_i = \begin{cases} S_i(1) & \text{if } D_i = 1, \\ S_i(0) & \text{if } D_i = 0. \end{cases}$$

- A hypothetical example: D_i indicates whether country i has a high ethnic diversity, S_i represents whether the country is developed, and Y_i is the frequency of civil conflicts.
- Suppose D_i is randomly assigned, hence

$$D_i \perp \{Y_i(0), Y_i(1), S_i(0), S_i(1)\}.$$

- Then,

$$\begin{aligned} & E[Y_i|D_i = 1, S_i = 1] - E[Y_i|D_i = 0, S_i = 1] \\ =& E[Y_i(1)|D_i = 1, S_i(1) = 1] - E[Y_i(0)|D_i = 0, S_i(0) = 1] \\ =& E[Y_i(1)|S_i(1) = 1] - E[Y_i(0)|S_i(0) = 1]. \end{aligned}$$

- We are making comparisons between two different sets of countries.

Bias amplification

- Controlling for more covariates sometimes results in undesirable consequences.
- Suppose X_i is significantly correlated with D_i but has little influence on Y_i.
- Controlling for X_i reduces the variation of D_i and increases the estimate's standard error.
- If X_i is positively correlated with Y_i and an unobservable confounder U_i is negatively correlated with Y_i, then ignoring X_i may offset the impact of U_i.
- Adding more control variables may cause bias amplification (Middleton et al. 2016).

Collider bias

- Why cannot GRE grade predict the achievement of PhD students?
- Why aren't smaller countries more likely to lose in wars?
- We are implicitly conditioning on a variable U, known as a collider, in these analyses:

$$X_1 \to U \leftarrow X_2.$$

- U is admission into the PhD program, or engagement in wars:
- Here X_1 is GRE grade/size of the country, and X_2 could be research experience/number of allies.
- If you are admitted into the program with a low GRE grade, your research experience might be better than average.
- If a small country is engaged in a war, it must be more prepared than larger countries.
- Essentially, conditioning on U leads to a biased sample, hence it is also known as the "sample selection bias."

Evaluate unconfoundedness

- It is impossible to directly test the assumption of unconfoundedness as it involves the joint distribution of $(Y_i(0), Y_i(1))$.
- But there are indirect ways to do so.
- The most common approach is to use placebo tests.
- Suppose there are some variables which are not supposed to be affected by the treatment, we can estimate the effect on them using the same estimator.
- Significant results would suggest the violation of the assumption.
- Or we can estimate the effect generated by a variable which should not affect the outcome.

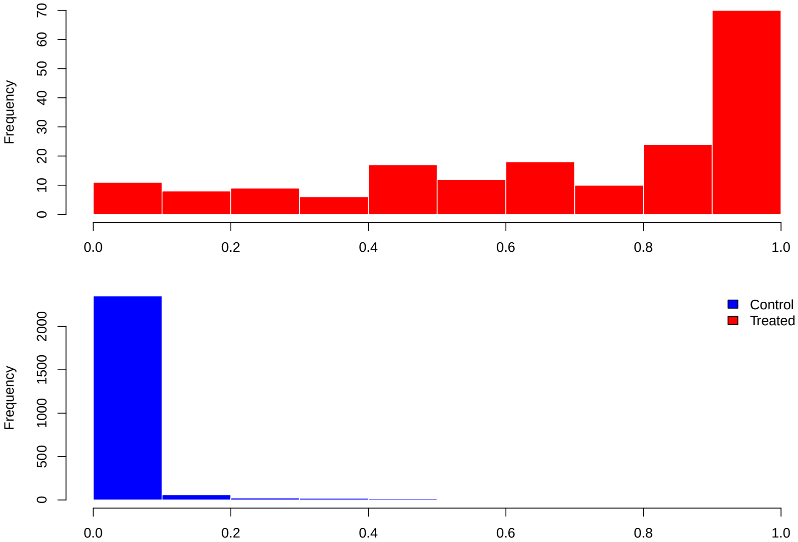
Evaluate unconfoundedness

- For example, if there is a new policy to encourage the school attendance of female students, we should not find an effect on male students.
- We should not find an effect on women who have finished school either.
- Similarly, the school attendance rate of women may not be affected by a policy that regulates gas price.
- The former is known as a placebo outcome, and the latter a placebo treatment.
- Note that the placebo outcome should be a post-treatment variable.
- Otherwise, you may want to control the variable in the analysis.
- A proper placebo test requires knowledge on the context we study.

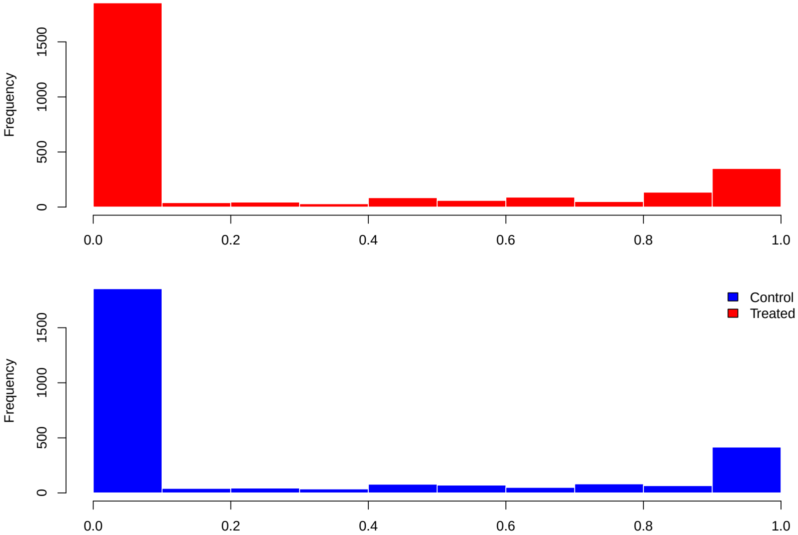
Consequences of weak positivity

- In theory, positivity is satisfied if $0 < P(D_i|\mathbf{X}_i) < 1$.
- But our methods will perform poorly if $P(D_i|\mathbf{X}_i)$ can be very close to 0 or 1.
- Khan and Tamer (2010) show that root-N consistent estimator may not exist in this case.
- That's why we usually write $\varepsilon < P(D_i|\mathbf{X}_i) < 1 - \varepsilon$ for some $0 < \varepsilon < 1$.
- Rothe (2017) argues that the confidence intervals may have poor coverage when ε is close to zero.
- It is necessary to examine the distribution of propensity scores across the two groups.

Evaluate positivity



Evaluate positivity



Sensitivity analysis

- The basic idea: how influential unobservable confounders have to be to drive the estimate insignificant/zero?
- Remember that confounders must be correlated with both D and Y.
- We can vary the magnitude of the two correlations and check how the estimate would change.
- To find a benchmark, we calculate the correlations of some observable confounders with D and Y.
- Methods differ in their assumptions on the DGP.
- It was motivated by Fisher's questioning on the causal relationship between smoking and lung cancer (Cornfield et al. 1959).
- Earlier works are built upon parametric assumptions (Rosenbaum and Rubin 1983; Imbens 2003) but now we can do better.

An omitted variable bias perspective

- Cinelli and Hazlett (2020) motivate their method from the perspective of the omitted variable bias in regression.
- Suppose the true model is $Y_i = \tau D_i + \mathbf{X}_i' \beta + \gamma U_i + \varepsilon_i$.
- But U is unobservable to the researcher.
- The model we estimate is $Y_i = \tau_s D + \mathbf{X}_i' \beta_s + \nu_i$.
- Let's use $V^{\perp \mathbf{X}}$ to denote the regression residual from estimating variable V on $\mathbf{X}$, then

$$
\begin{aligned}
\hat{\tau}_s =& \frac{Cov(D^{\perp \mathbf{X}}, Y^{\perp \mathbf{X}})}{Var(D^{\perp \mathbf{X}})} \\
=& \frac{Cov(D^{\perp \mathbf{X}}, \hat{\tau} D^{\perp \mathbf{X}} + \hat{\gamma} U^{\perp \mathbf{X}})}{Var(D^{\perp \mathbf{X}})} \\
=& \hat{\tau} + \hat{\gamma} \frac{Cov(D^{\perp \mathbf{X}}, U^{\perp \mathbf{X}})}{Var(D^{\perp \mathbf{X}})} \\
=& \hat{\tau} + \hat{\gamma} \hat{\delta}
\end{aligned}
$$

An omitted variable bias perspective

- The difference between the correct estimate $\hat{\tau}$ and the actual estimate $\hat{\tau}_s$ consists of two parts:
  1. $\hat{\gamma}$: the impact of the unobservable covariate on the outcome,
  2. $\hat{\delta}$: the imbalance of the unobservable between the two groups.
- Essentially, the estimate is robust to model misspecification when both Y and D can be largely explained by the observable covariates.
- We can rely on R^2 to measure the explanatory power of any covariates.

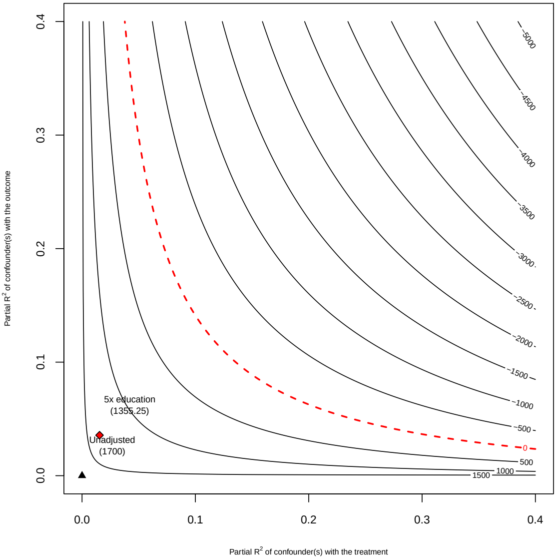
An omitted variable bias perspective

- Cinelli and Hazlett (2020) show that

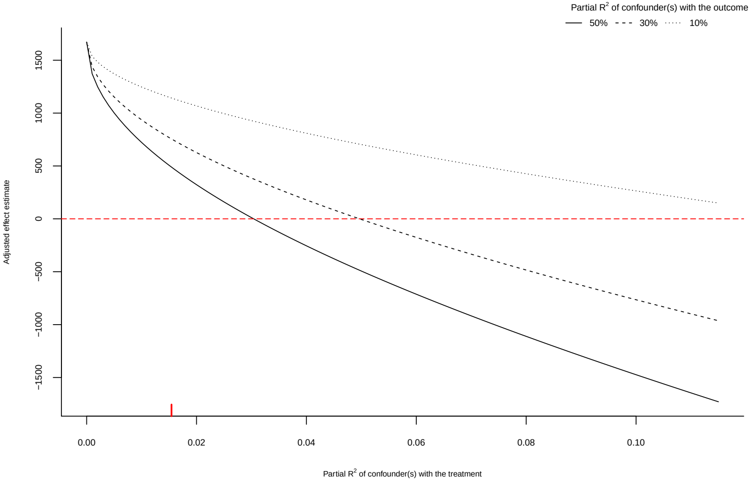
$$|\hat{\gamma}\hat{\delta}| = \sqrt{\frac{R^2_{Y\sim U|\mathbf{X},D} R^2_{D\sim U|\mathbf{X}}}{1 - R^2_{D\sim U|\mathbf{X}}}} \left(\frac{sd(Y^{\perp \mathbf{X},D})}{sd(D^{\perp \mathbf{X}})} \right)$$

- Model misspecification is not dependent on the sample size.
- We vary the values of $R^2_{Y\sim U|\mathbf{X},D}$ and $R^2_{D\sim U|\mathbf{X}}$ to see how the estimate changes.
- It is straightforward to generalize the method to more complicated models.
- E.g., correct vs. misspecified influence functions (Chernozhukov et al. 2022).

An omitted variable bias perspective: application



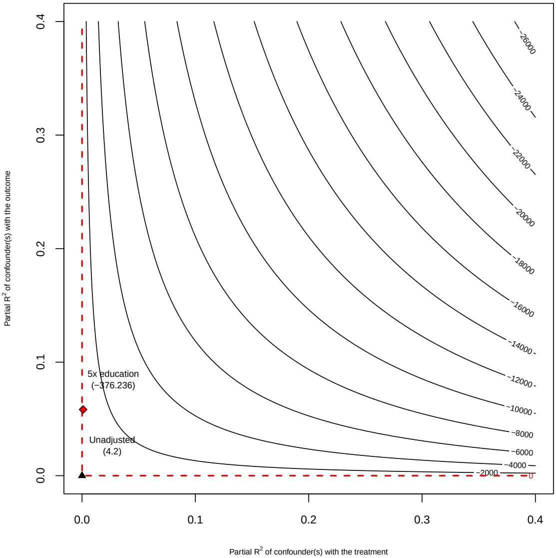
An omitted variable bias perspective: application



An omitted variable bias perspective: application

```
## Sensitivity Analysis to Unobserved Confounding
##
## Model Formula: re78 ~ treat + age + education + black +
##     nodegree + re74 + re75 + u74 + u75
##
## Null hypothesis: q = 1 and reduce = TRUE
## -- This means we are considering biases that reduce the
## -- The null hypothesis deemed problematic is H0:tau = 0
##
## Unadjusted Estimates of 'treat':
##   Coef. estimate: 1670.71
##   Standard Error: 641.1323
##   t-value (H0:tau = 0): 2.6059
##
## Sensitivity Statistics:
##   Partial R2 of treatment with outcome: 0.0154
##   Robustness Value, q = 1: 0.1176
##   Robustness Value, q = 1, alpha = 0.05: 0.0302
```

An omitted variable bias perspective: application



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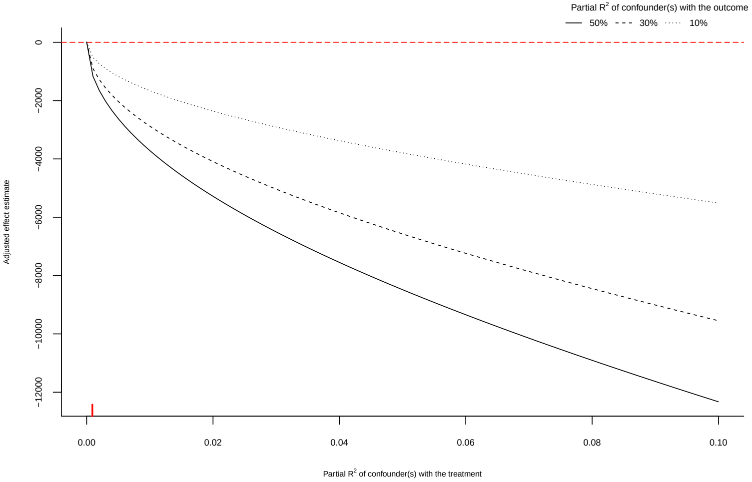

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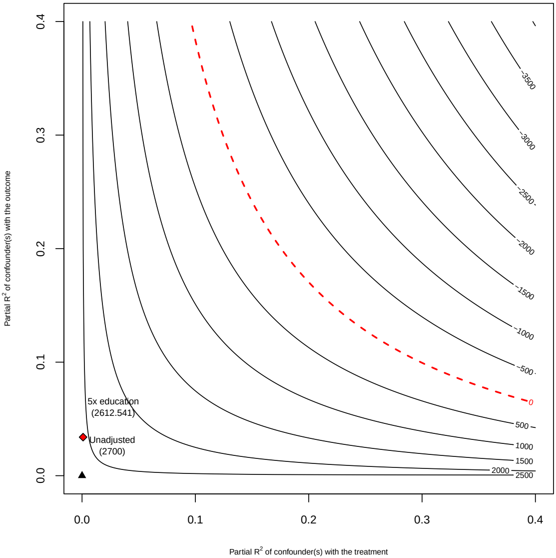

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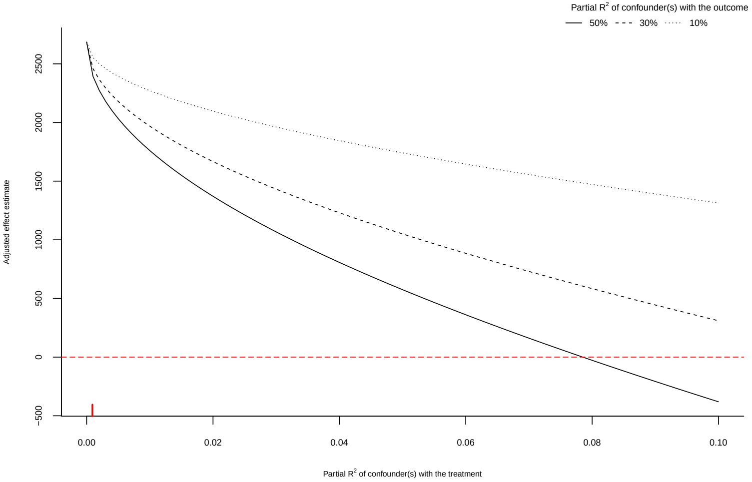

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