

TSCS Data Analysis

Ye Wang

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- ▶ We then introduce several approaches to fix the problems, such as counterfactual estimation.
- ▶ We next review available methods under sequential ignorability.
- ▶ We conclude by discussing the idea of manifold learning and how to deal with interference in TSCS data (hopefully!).

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 $D_i \perp Y_i(D_i) | \mathbf{X}_i, 0 < P(D_i = 1 | \mathbf{X}_i) < 1.$
- ▶ Now we can relax the assumption along two possible directions (based on your belief).

What is unique about TSCS data?

- ▶ Strict exogeneity: $D_{it} \perp Y_{is}(D_{is}) | \mathbf{X}_i^{1:t}, \mathbf{U}_i^{1:t}$, where $\mathbf{X}_i^{1:t}$ the history of observable confounders and $\mathbf{U}_i^{1:t}$ the history of unobservable confounders.

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- ▶ Intuitively, their values do not hinge on the value of the outcome in history.
- ▶ We always assume that SUTVA holds and there is no reversal causality (future treatments won't affect past outcomes).

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- ▶ You will be vaccinated on Feb. 15 with probability 0.72 if you are an old Asian male who loves tequila and that day's temperature is above zero.
- ▶ The dataset available to the analyst only includes each unit's observable attributes, health status, treatment status, and the vaccination date.

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- ▶ Under strict exogeneity, we have the problem of omitted variables as part of the confounders ($\mathbf{U}_i^{1:T}$) is also unknown.
- ▶ To eliminate the influence of the unobservables, we need stronger assumptions on the DGP, such that the values of $\mathbf{U}_i^{1:T}$ can be learned from other variables.

Estimation under strict exogeneity

- ▶ Note that the strict exogeneity assumption is satisfied by the following outcome model:

$$Y_{it} = g_{it}(D_{it}, \mathbf{X}_i^{1:t}, \mathbf{U}_i^{1:t}) + \varepsilon_{it}$$
$$\mathbb{E}[\varepsilon_{is} | D_{it}, \mathbf{X}_{it}, \mathbf{U}_{it}] = 0,$$

which is too general for identification.

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- ▶ For example, we can assume that $h_{it}(\mathbf{U}_{it}) = \mu + \alpha_i + \xi_t$, then

$$Y_{it} = \mu + \delta_{it}D_{it} + \mathbf{X}_{it}\beta + \alpha_i + \xi_t + \varepsilon_{it}.$$

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- ▶ Now, the assumption of strict exogeneity becomes:
 $\mathbb{E}[\varepsilon_{is} | D_{it}, \mathbf{X}_{it}, \alpha_i, \xi_t] = 0$ for any s .

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Estimation under strict exogeneity

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- ▶ We no longer have consistent estimates if treatment effects are heterogeneous.
- ▶ There are several solutions to the problem.
- ▶ It is easier to fix the problem under the DID setting (Strezhnev, 2017).
- ▶ Otherwise, we need to modify the estimand or impose stronger assumptions.

DIDM

- ▶ Chaisemartin and D'Haultfœuille (2020) advocate that we should focus only on the instant effects generated by the treatment and estimate

$$\frac{1}{N^*} \sum_{t \geq 2, D_{it} \neq D_{i,t-1}} \delta_{it}$$

where $N^* = \sum_{i=1}^N \sum_{t=1}^T \mathbf{1}\{t \geq 2, D_{it} \neq D_{i,t-1}\}$.

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- ▶ This estimand is well-defined and can be consistently estimated.
- ▶ Yet it throws away a ton of information as well thus is not very efficient.
- ▶ There are more efficient approaches which require stronger assumptions on model specification.

Counterfactual estimation

- ▶ Xu (2017) and Liu, Wang, and Xu (2020) combine factor models with the Neyman-Rubin framework

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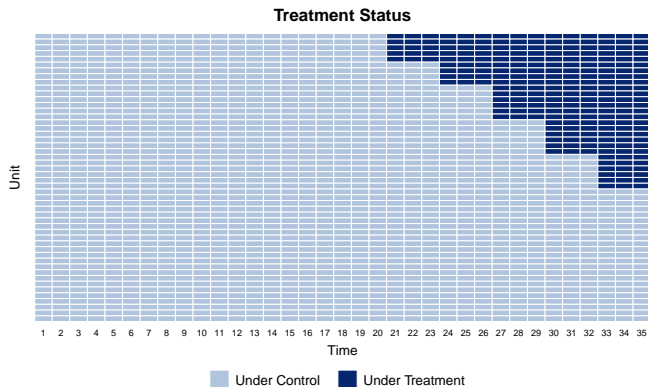
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- ▶ Clearly, $\hat{\delta}_{it} = Y_{it} - \hat{Y}_{it}(0)$ and

$$\widehat{ATT} = \frac{1}{|\mathcal{M}|} \sum_{(i,t) \in \mathcal{M}} \hat{\delta}_{it}$$

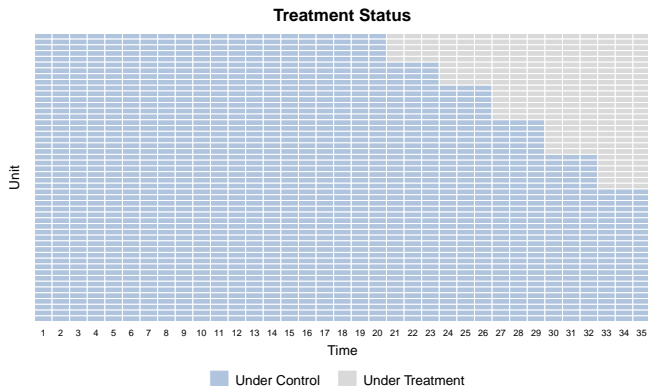
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- ▶ In a TSCS setting, treat $Y(1)$ as missing data



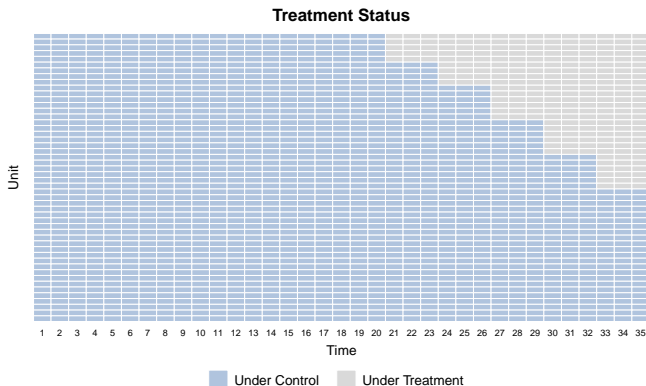
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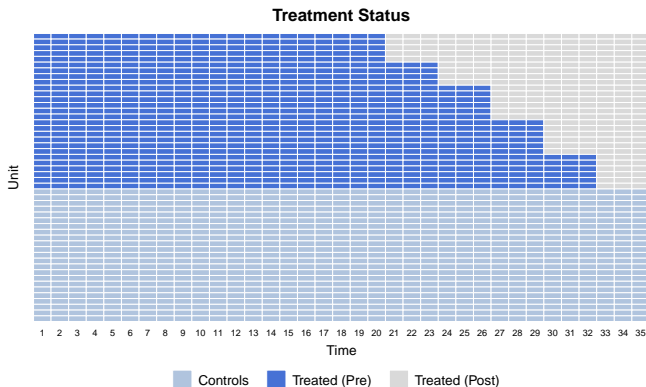
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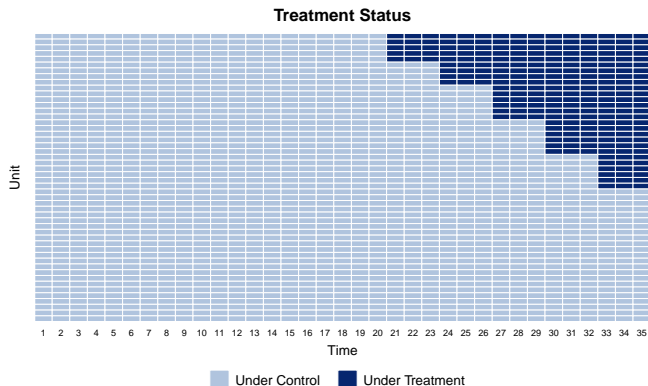
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- ▶ Predict $Y(0)$ based on an outcome model (FE or IFE)
- ▶ (Use pre-treatment data for model selection)
- ▶ Estimate ATT by averaging differences between $Y(1)$ and $\hat{Y}(0)$



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 - ▶ Observable and unobservable confounders are separable.
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 - ▶ Unobservable confounders have a low-dimensional decomposition.
- ▶ It also requires strict exogeneity, thus excludes temporal interference.

Counterfactual estimation

- ▶ One advantage of the framework is that testing the identification assumption becomes straightforward.

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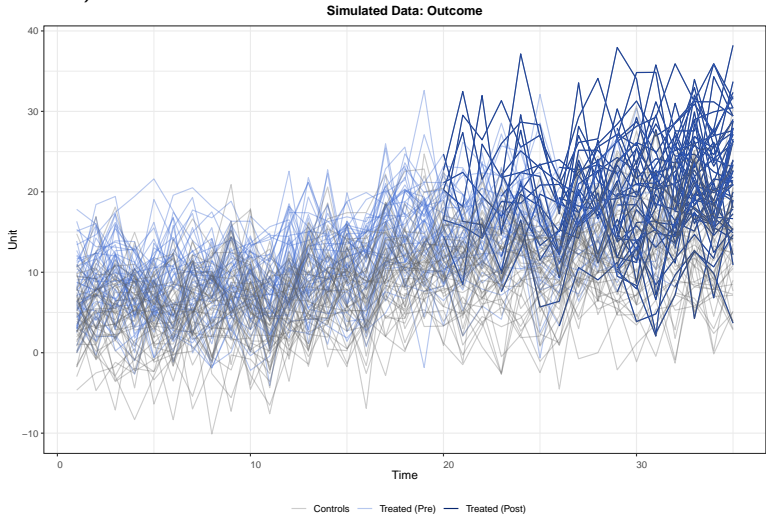
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- ▶ A plot of dynamic effects.
- ▶ A placebo test: estimate treatment effects before the treatment's onset and test their significance.
- ▶ An equivalence test: test whether all the pre-treatment ATTs are equal to zero using TOST.

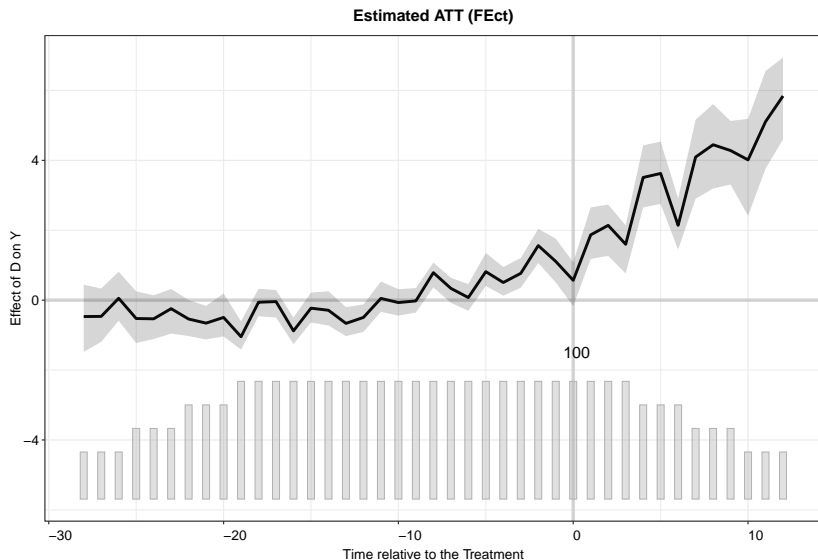
Example

- ▶ Let's consider a DGP with two factors (hence FEct will be biased).

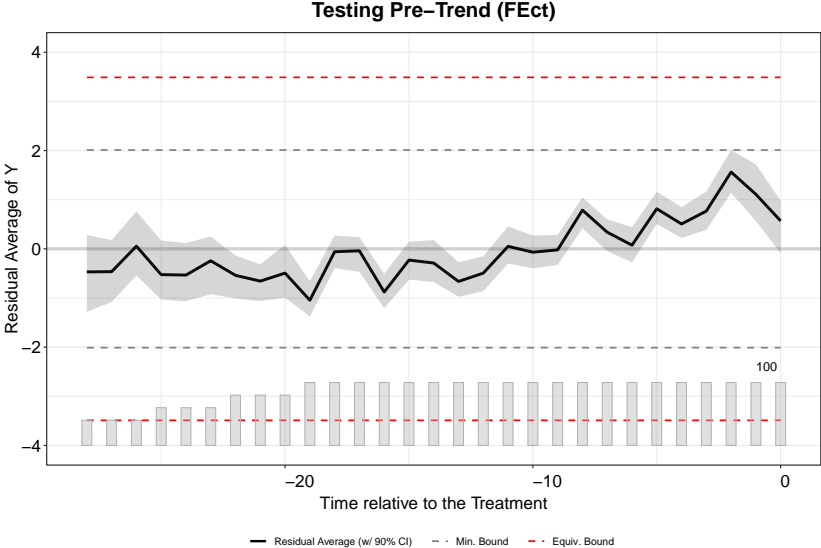


Example

Bootstrapping for uncertainties100 runs

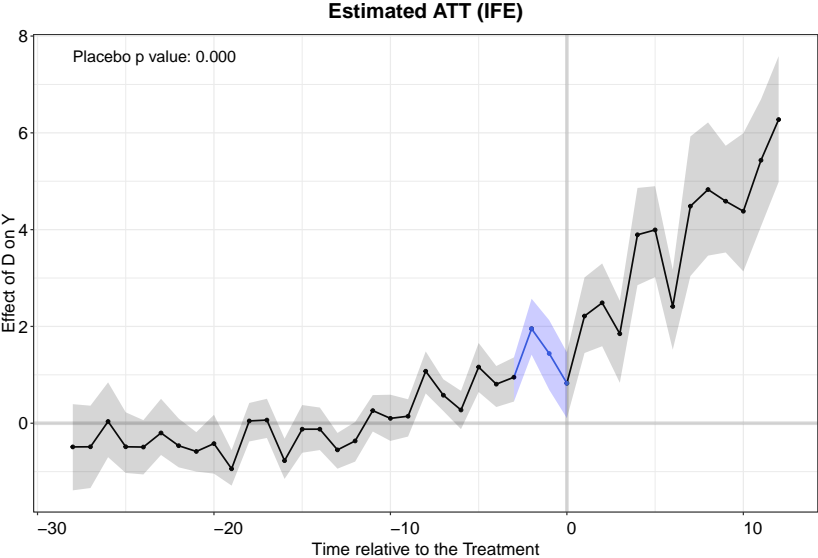


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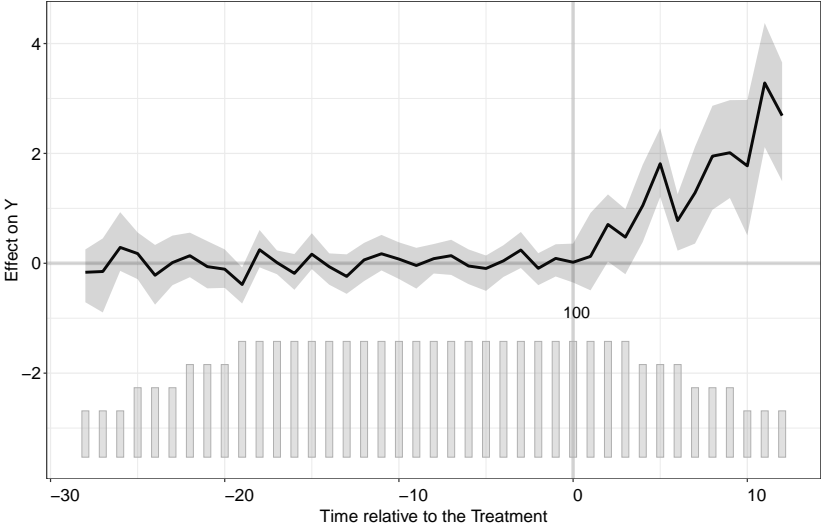
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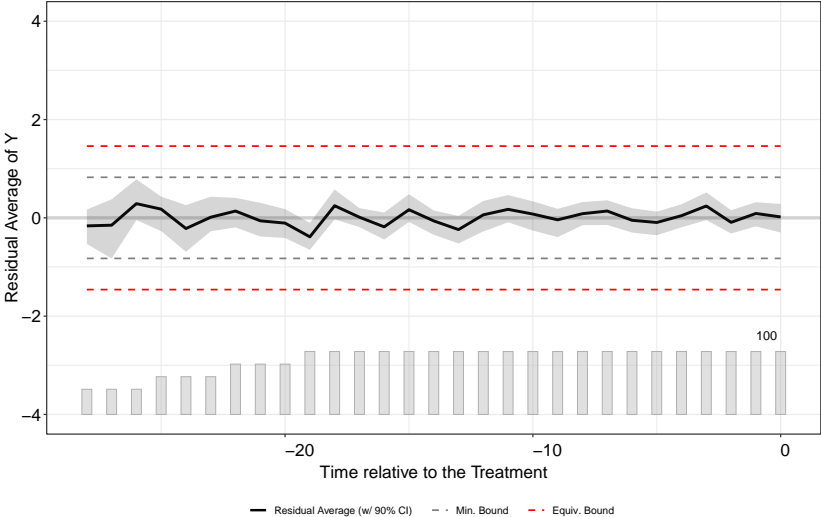
Example

Estimated ATT (IFect)



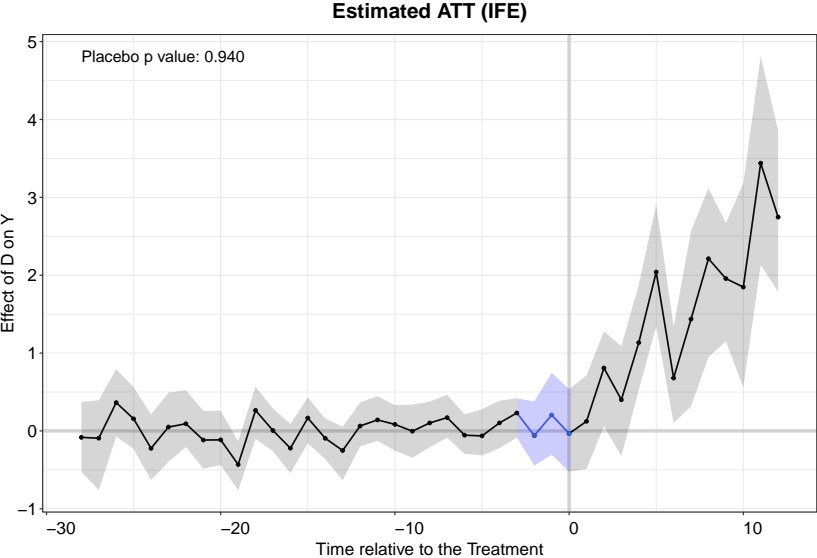
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Testing Pre-Trend (IFect)



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- ▶ The lagged dependent variable does solve a lot of problems, as the time trend is usually the most important confounder.
- ▶ Modern approach: IPTW estimators (MSMs) and doubly robust estimators.
- ▶ Notice that the propensity score for unit i in period t has the form of $P(D_{it} | \mathbf{Y}_i^{1:(t-1)}, \mathbf{X}_i^{1:t})$.

Weighting under sequential ignorability

- ▶ Suppose there is no interference and we are interested in the contemporary effect generated by the treatment:

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- ▶ We can select a model to estimate the propensity score $e_{it} = P(D_{it} = 1 | \mathbf{Y}_i^{1:(t-1)}, \mathbf{X}_i^{1:t})$ and apply the IPTW estimator:

$$\hat{\tau}_t = \frac{1}{N} \sum_{i=1}^N \frac{D_{it} Y_{it}}{e_{it}} - \frac{1}{N} \sum_{i=1}^N \frac{(1 - D_{it}) Y_{it}}{1 - e_{it}}.$$

Weighting under sequential ignorability

- ▶ Suppose there is no interference and we are interested in the contemporary effect generated by the treatment:

$$\tau_t = \frac{1}{N} \sum_{i=1}^N \tau_{it}, \text{ where } \tau_{it} = Y_{it}(1) - Y_{it}(0)$$

- ▶ Remember that sequential ignorability means $D_{it} \perp Y_{it}(D_{it}) | \mathbf{Y}_i^{1:(t-1)}, \mathbf{X}_i^{1:t}$ for any t .
- ▶ We can select a model to estimate the propensity score $e_{it} = P(D_{it} = 1 | \mathbf{Y}_i^{1:(t-1)}, \mathbf{X}_i^{1:t})$ and apply the IPTW estimator:

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- ▶ It is just another Horvitz-Thompson estimator.

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- ▶ But when T is large, balancing the entire history can be challenging in practice.
- ▶ Hazlett and Xu (2018) suggest that we may instead balance kernels of the history.
- ▶ The implicit assumption is that the distribution of the outcome history can be well approximated by its kernels.

Balancing under sequential ignorability

- ▶ We search for a group of positive weights $\{w_i\}$ which sum to one such that

$$\frac{1}{|\mathcal{T}|} \sum_{i \in \mathcal{T}} \mathbf{K}_i = \sum_{i \in \mathcal{C}} w_i \mathbf{K}_i$$

where $\mathbf{K}_i = (k(\mathbf{Y}_i, \mathbf{Y}_1), k(\mathbf{Y}_i, \mathbf{Y}_2), \dots, k(\mathbf{Y}_i, \mathbf{Y}_N))$, \mathbf{Y}_i is the pre-treatment outcome history of unit i , and

$$k(\mathbf{Y}_i, \mathbf{Y}_j) = e^{-\|\mathbf{Y}_i - \mathbf{Y}_j\|^2/h}.$$

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- ▶ Next, we estimate the treatment effect on the treated in each period

$$\hat{\tau}_t = \frac{1}{|\mathcal{T}|} \sum_{i \in \mathcal{T}} Y_{it} - \sum_{i \in \mathcal{C}} w_i Y_{it}$$

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- ▶ Kim, Imai, and Wang (2019) show that we can do the same via matching.

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- ▶ $S_i = \frac{1}{T} \sum_{t=1}^T D_{it}$ is a sufficient statistic for α_i .

Manifold learning

- ▶ Arkhangelsky and Imbens (2019) further illustrate that under this assumption, all we need to do is to find a group of weights (again!) such that:

$$\begin{aligned} \{\hat{w}_{it}\} &= \arg \min \frac{1}{NT} \sum_{i=1}^N \sum_{t=1}^T \hat{w}_{it}^2, \\ \text{s.t. } &\frac{1}{NT} \sum_{i=1}^N \sum_{t=1}^T \hat{w}_{it} D_{it} = 1, \quad \frac{1}{N} \sum_{i=1}^N \hat{w}_{it} = 0, \quad \frac{1}{T} \sum_{t=1}^T \hat{w}_{it} = 0, \\ &\frac{1}{NT} \sum_{i=1}^N \sum_{t=1}^T \hat{w}_{it} \psi_{it}(S_i) = 0, \quad \hat{w}_{it} D_{it} \geq 0. \end{aligned}$$

- ▶ Then, the estimate of the ATE,

$$\hat{\tau} = \frac{1}{NT} \sum_{i=1}^N \sum_{t=1}^T \hat{w}_{it} Y_{it}$$

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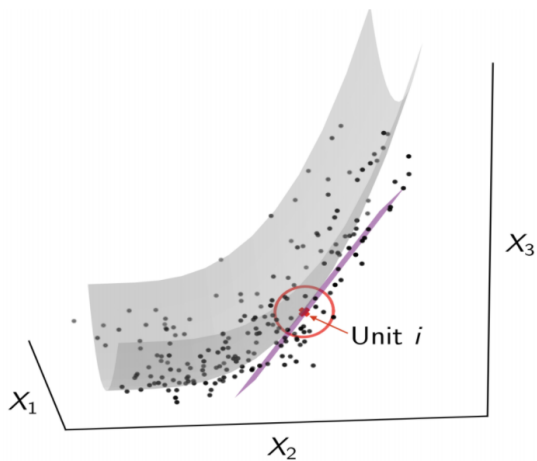
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- ▶ To estimate λ_i , we first match each unit i to K nearest neighbors.
- ▶ Next, we estimate λ_i via PCA on the $K + 1$ outcome histories.

Manifold learning

Figure 2: Local Tangent Space Approximation



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- ▶ Treatment assignments are usually temporally dependent in panel data.
- ▶ Now, past treatments become confounders.
- ▶ We have to control their influence under either assumption.

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- ▶ We can still estimate the quantity defined by Chaisemartin and D'Haultfœuille (2020), but nothing else.
- ▶ Sequential ignorability is the more suitable assumption if we have temporal interference

$$D_{it} \perp Y_{it}(D_{it}) | \mathbf{Y}_i^{1:(t-1)}, \mathbf{D}_i^{1:(t-1)}, \mathbf{X}_i^{1:t}.$$

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- ▶ We can obtain valid estimate for the history's aggregated effect after weighting observations properly.
- ▶ Now,

$$P(\mathbf{D}_i^{s:t} = \mathbf{d}^{s:t}) = \prod_{s'=s}^t P(D_{is'} | \mathbf{D}_i^{1:(s'-1)}, \mathbf{Y}_i^{1:(s'-1)}, \mathbf{X}_i^{1:s'}).$$

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- ▶ Wang (2020) shows that the limit of the DID estimator has no substantive interpretation under spatial interference.
- ▶ Intuitively, the control group is contaminated and we cannot eliminate the bias via outcome adjustment.
- ▶ Yet under sequential ignorability, we can combine the idea in Aronow, Samii, and Wang (2020) and MSMs to generate consistent estimates for both the direct and the indirect effects.

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