Heterogeneous Treatment Effects I

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Linear Methods in Causal Inference
POLI784

Review

- We discussed the causal interpretation of the OLS estimator in the previous class.
- In randomized experiments, the OLS estimator equals the Hajek estimator.
- The HC2 variance estimator equals the Neyman variance estimator.
- We may use regression adjustment to control for covariates and enhance the efficiency of the OLS estimator.
- This is justified by the FWL theorem when the model specification is correct.
- Otherwise, we can rely on Lin's regression to ensure the increase in efficiency.

From ATE to CATE

- Sometimes we want to know the average treatment effect on a sub-population:

$$\tau(\mathbf{x}) = E[\tau_i | \mathbf{X} = \mathbf{x}].$$

- This is known as the conditional average treatment effect (CATE).
- It allows us to see how the effects vary within the population and helps researchers to design more personalized policy or medicine.
- Note that $\mathbf{X}$ should not be affected by the treatment.
- It is sometimes called the moderator.

From CATE to external validity

- CATE is closely connected to the external validity of a study.
- With a representative sample, a consistent estimate for SATE is consistent for PATE as well.
- But this is rarely the case.
- We want to know some general laws of human behavior.
- But the sample often comes from one country or even one county.
- How do we generalize our estimate obtained from one sample to the population?

External validity

- We need to understand how SATE differs from PATE.
- One possibility: it is completely driven by the difference in demographic composition.
- Suppose the only variable that affects the effect's size is age and our experiment is conducted in a county with more senior people.
- To generalize the conclusion to the whole country, we just need to reweigh our sample with the proportion of senior residents in America.
- A more severe issue is known as the site-selection bias.
- There are unobservable factors that are correlated with both the effects and where the experiment is implemented.
- It is an open question in the literature.

Estimate CATE

- The remaining question: how do we estimate the CATE?
- If $\mathbf{X}$ only includes binary variables, we can estimate the ATE conditional on each value of $\mathbf{X}$.
- It is equivalent to estimating a regression model with an interaction term:

$$Y_i = \mu + \tau D_i + \beta X_i + \delta D_i * X_i + \varepsilon_i.$$

- Such a model is "saturated" as it covers all the combinations of D_i and X_i.
- The estimated effect of D_i equals $\hat{\tau}$ if $X_i = 0$ and $\hat{\tau} + \hat{\delta}$ if $X_i = 1$.

Estimate CATE

- Note that X_i is not randomly assigned, hence the difference between $\tau(1)$ and $\tau(0)$ does not have a causal interpretation.
- E.g., we cannot say "turning old increases the effect by 20%."
- It is different from

$$Y_i = \mu + \tau D_{1i} + \beta D_{2i} + \delta D_{1i} * D_{2i} + \varepsilon_i,$$

  where both D_1 and D_2 are randomly assigned.
- If interested in the interaction effect, we have to control for confounders that affect X_i.

Estimate CATE

- If **X** includes continuous variables, the convention is to fit the same regression model.
- We have learned that Lin's regression is the better approach:

$$Y_i = \mu + \tau D_i + (X_i - \bar{X})\beta + \delta D_i * (X_i - \bar{X}) + \varepsilon_i.$$

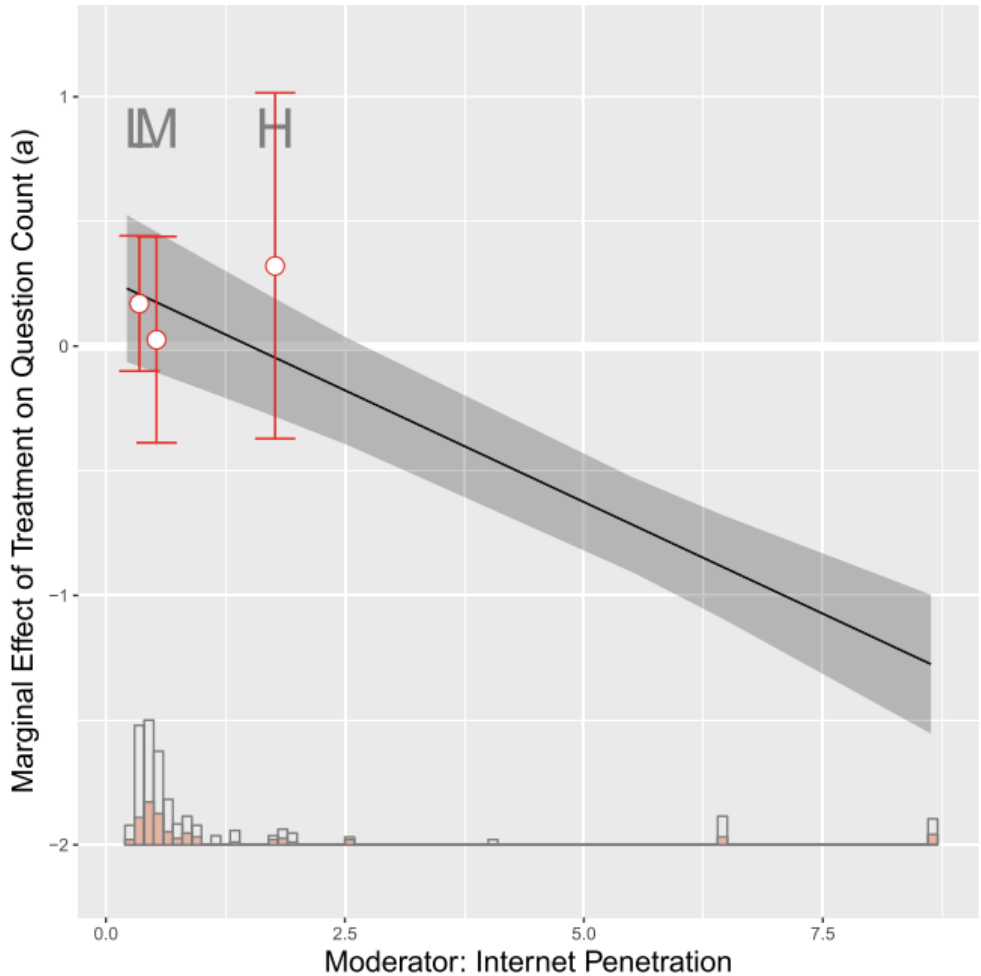
- The estimated moderator effect equals $\hat{\tau} + \hat{\delta}(X_i - \bar{X})$, a linear function of X.
- There is no guarantee that this linear relationship holds.

Caveats of interaction models

- Consider the following application in Malesky, Schuler, and Tran (2012).
- It is an experiment implemented in Vietnam.
- Treatment: an online profile for randomly selected legislators that documents their performance.
- Outcome: questions a legislator asked in Congress.
- Their ATE estimate is not significant.
- But the interaction model shows that the effect is significant in regions where the Internet penetration rate is high.

Caveats of interaction models

- Hainmueller, Mummolo, and Xu (2019) show that the estimate is entirely driven by certain regions.



Caveats of interaction models

- This example illuminates the problems of relying on linear models.
- The predictions can be very inaccurate if the true pattern is not quite linear.
- The results can be influenced by a few observations in the sample.
- It is because regression is a global model.

Estimate the CATE flexibly

- Remember that we want to estimate

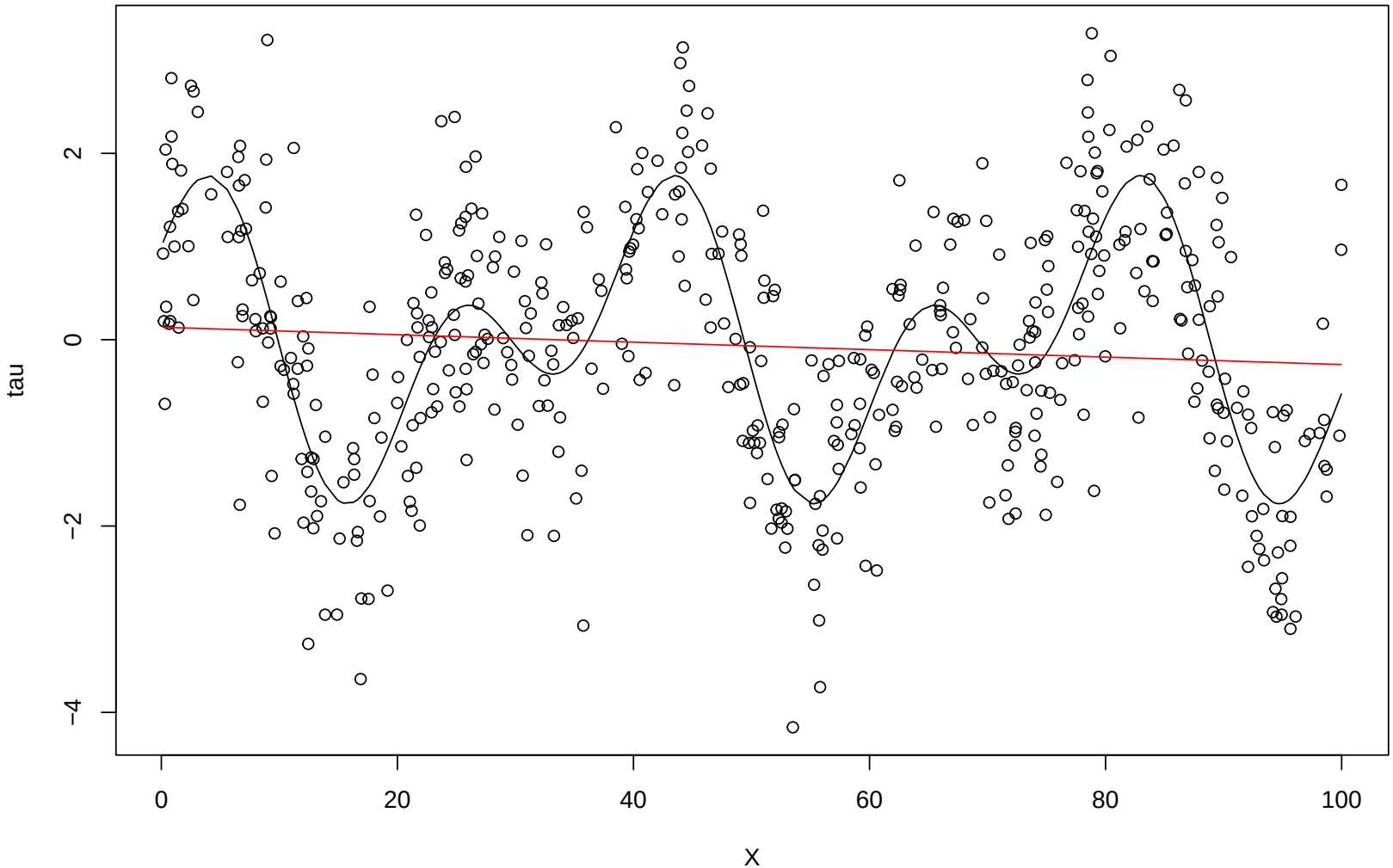
$$\tau(x) = E[\tau_i | X_i = x].$$

  without assuming a linear relationship.
- Let's first assume we know the value of each τ_i.
- It becomes a problem of estimating the conditional expectation of a variable.
- This is a prediction problem rather than a causal inference problem.

Estimate conditional expectation

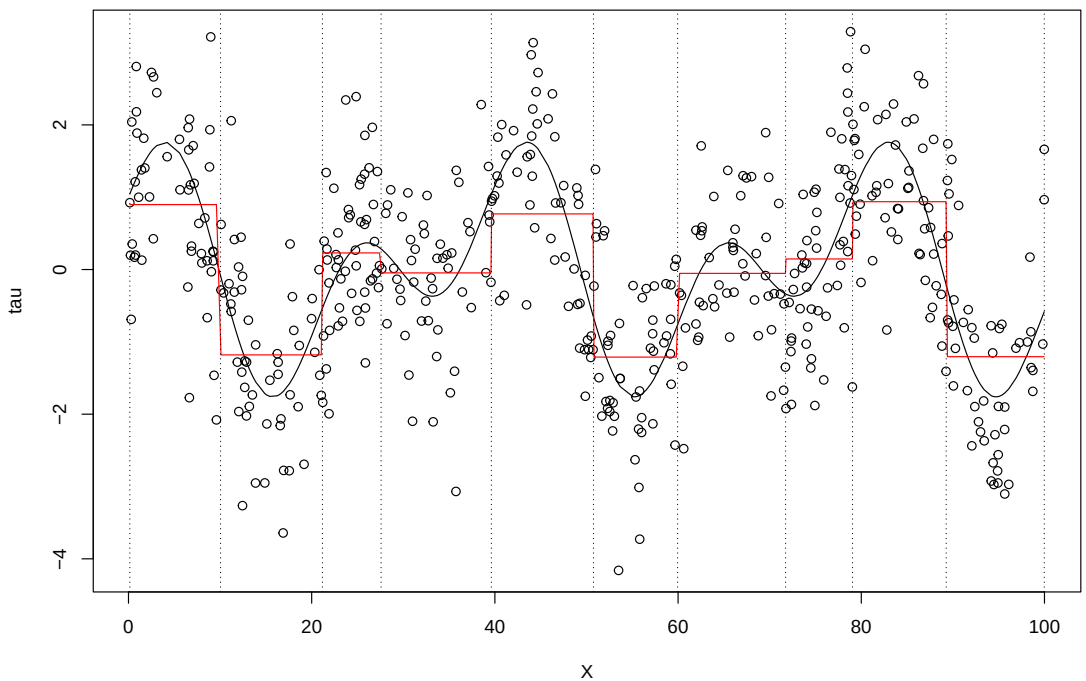
- Later we discuss how to deal with the problem of estimating the CATE using similar techniques.
- We have learn the regression approach, which assumes that $\tau(x) = \beta x$.
- Instead of linearity, let's only assume the smoothness of $\tau(x)$.
- This is much weaker and satisfied in many scenarios.
- A common form of such an assumption is the sth order derivative of $\tau(x)$ exists.

Estimate conditional expectation



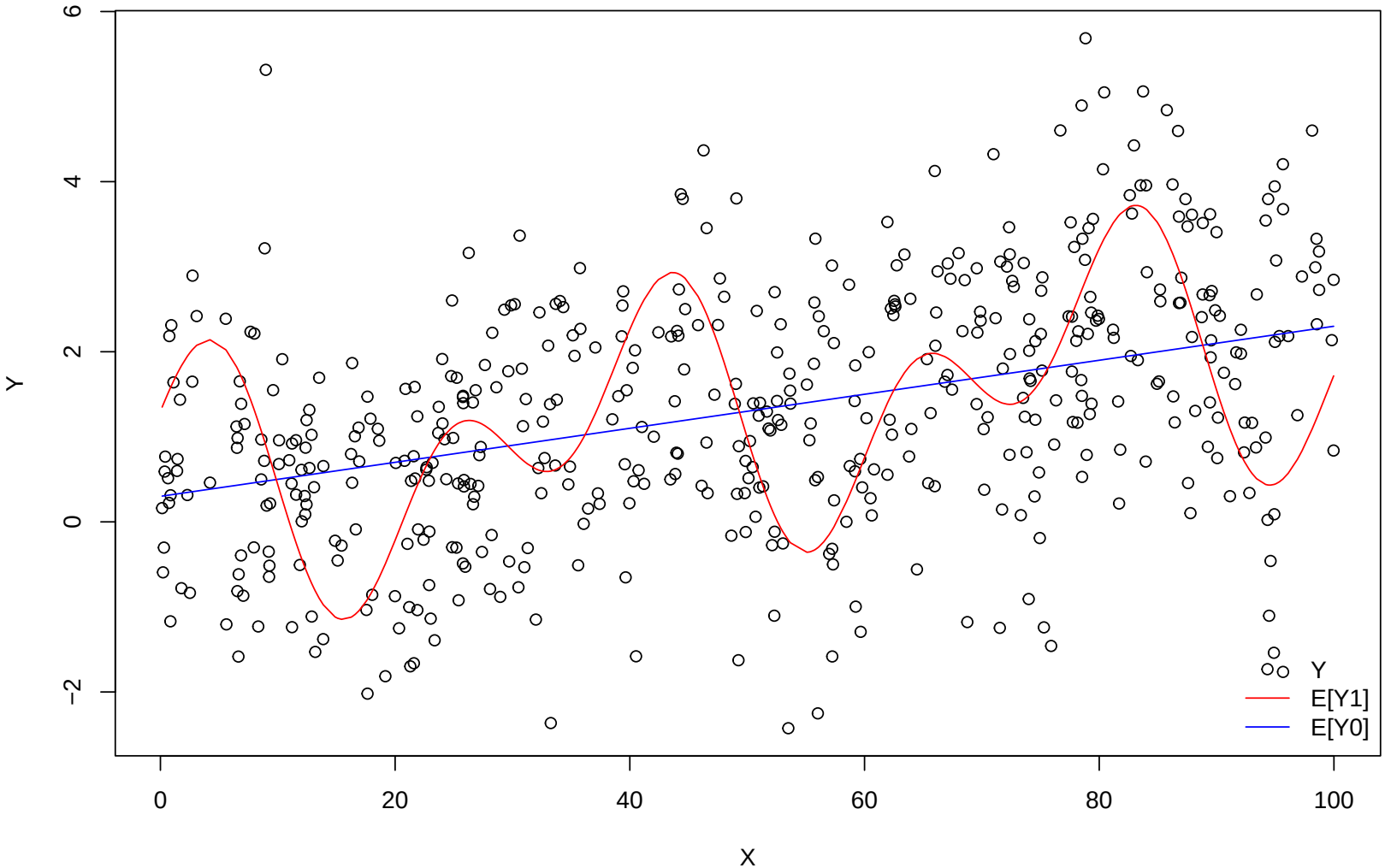
The binscatter estimator

- Smoothness means that if x' is close to x, then $\tau(x')$ is close to $\tau(x)$.
- Therefore, we can estimate $\tau(x)$ using information from $\tau(x')$.
- A natural estimator is to divide the support of X into K bins and estimate $\tau(x)$ using the average of τ_i within each bin.

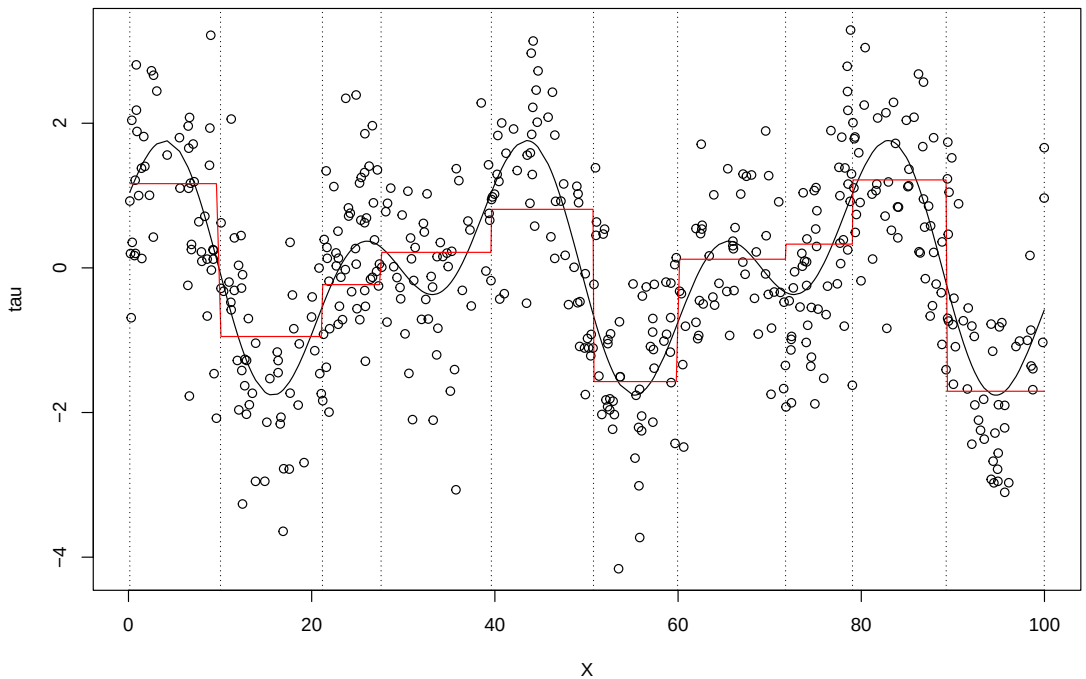


The binscatter estimator for the CATE

- With unknown τ_i, we apply the HT or HA estimator in each of the bins.

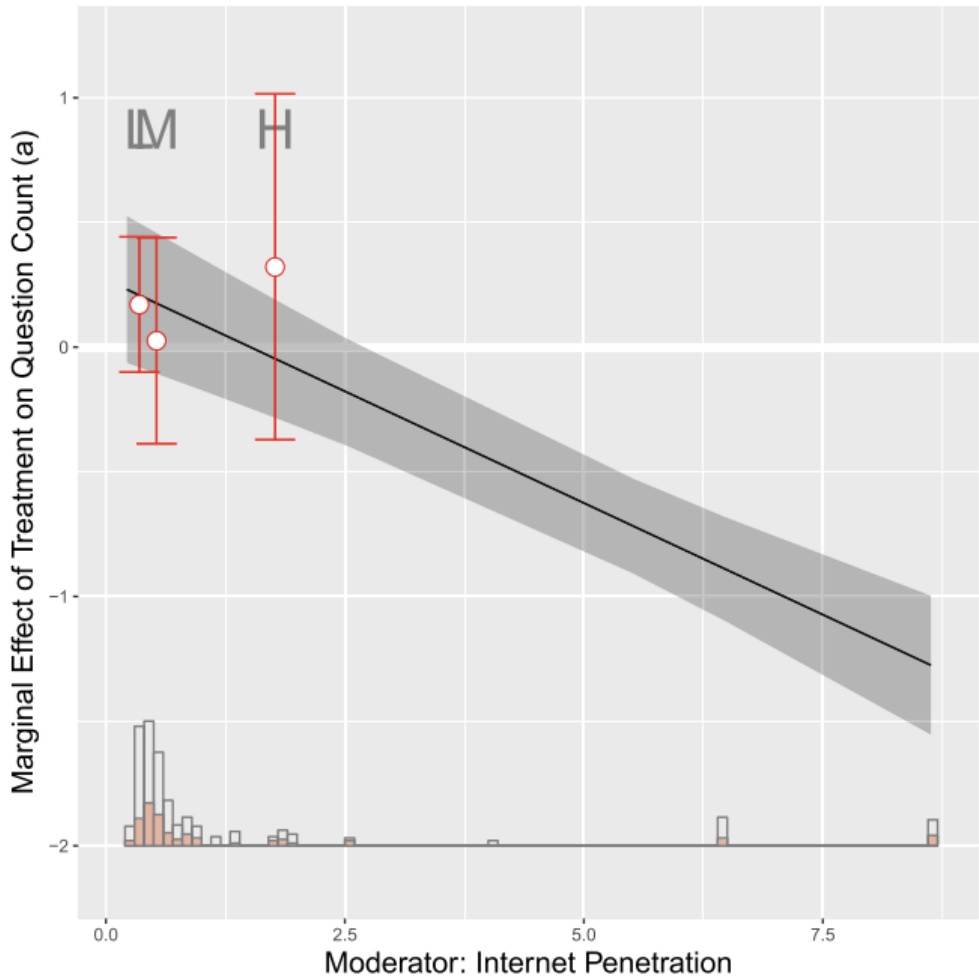


The binscatter estimator for the CATE



The binscatter estimator for the CATE

- Hainmueller, Mummolo, and Xu (2019) suggest that we use three bins.



- There are a lot of different choices (Cattaneo et al. 2019).
- Note that the estimator is clearly biased.

References I

Cattaneo, Matias D, Richard K Crump, Max H Farrell, and Yingjie Feng. 2019. "On Binscatter." *arXiv Preprint arXiv:1902.09608*.

Hainmueller, Jens, Jonathan Mummolo, and Yiqing Xu. 2019. "How Much Should We Trust Estimates from Multiplicative Interaction Models? Simple Tools to Improve Empirical Practice." *Political Analysis* 27 (2): 163–92.

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