Instrumental Variable I

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Linear Methods in Causal Inference
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Review

- We should be aware of the post-treatment bias, bias amplification, and the collider bias when selecting potential confounders.
- Placebo tests can be applied to evaluate the assumption of unconfoundedness.
- Even weak positivity has severe consequences.
- We can gauge the influence of unobservable confounders using sensitivity analysis.
- It shows how sensitive our estimates are to the correlations between the unobservable confounders with the outcome and the treatment.

Non-compliance

- So far, we have assumed that subjects in an experiment always comply with the treatment assignment.
- But this is often not the case.
- Patients want to get the treatment in medical experiments.
- People may not adopt the treatment due to social pressure.
- Non-compliance can occur in one group (one-sided) or both groups (two-sided).
- If so, treatment assignment Z_i will differ from the actual treatment status D_i.
- What quantity can we estimate in such a scenario? How do we estimate it?

Intention-to-treat effect

- Consider a classical randomized experiment with the outcome Y_i, the treatment assignment $Z_i \in \{0, 1\}$, and the treatment status $D_i \in \{0, 1\}$.
- Z_i is randomly assigned but D_i is not.
- We can still identify the intention-to-treat (ITT) effect defined as $\tau_{ITT} = E[Y_i|Z_i = 1] - E[Y_i|Z_i = 0]$.
- It can be estimated via the Horvitz-Thompson or the Hajek estimator.
- The ITT effect equals the ATE when there is perfect compliance.
- Whether this estimand makes sense depends on the context.
- Are we interested in the effect of a policy or its efficacy?

Local average treatment effect

- It is natural to assume that D_i's value can be affected by Z_i.
- We can define the potential outcomes for D_i:

$$D_i = \begin{cases} D_i(1) \text{ if } Z_i = 1, \\ D_i(0) \text{ if } Z_i = 0. \end{cases}$$

- There are four possibilities:

$$(D_i(0), D_i(1)) = \begin{cases} (1,1), \\ (0,1), \\ (0,0), \\ (1,0). \end{cases}$$

- We assume that these responses are decided by the nature of these units rather than the received assignment.
- E.g., Do you believe that there are microchips in the vaccine?

Local average treatment effect

- The efficacy of a treatment is decided by the effect on those who are encouraged to take it: $D_i(1) = 1 > D_i(0) = 0$.
- We call them "compliers" and the effect on them the local average treatment effect (LATE):

$$\tau_{LATE} = E[\tau_i | D_i(1) > D_i(0)].$$

- The LATE does not equal to the ATE and is defined on a sub-population.
- That's why it is "local."
- It is the expectation of τ_i conditional on the type of i.
- The type of a unit may differ across experiments but may not be affected by treatment assignment.

Principal strata

- There are three other possible types in the sample: always-takers ($D_i(1) = D_i(0) = 1$), never-takers ($D_i(1) = D_i(0) = 0$), and defiers ($D_i(1) = 0 < D_i(0) = 1$).
- It is impossible to identify the effect on the always-takers or never-takers (why?).
- We call these types "principal strata."
- This is a very general concept in causal inference.
- In medical trials, we have patients who can always survive over the experimental period, who can never survive, who can survive only after taking the treatment, and who cannot survive only after taking the treatment.
- In survey experiments, we have respondents who will always fill the survey, who will never fill the survey, who will fill the survey only if treated, and who will fill the survey only if under control.

Principal strata

- We cannot tell which type each unit belongs to as we cannot observe their responses under different assignments.
- The table below shows all the possibilities for each combination of (Z_i, D_i):

Values	$D_i = 0$	$D_i = 1$
$Z_i = 0$	Never-taker/Complier	Always-taker/Defier
$Z_i = 1$	Never-taker/Defier	Always-taker/Complier

- If you do not take vaccine when you are under control $(Z_i = 0, D_i = 0)$, you can be either a never-taker or a complier.

Assumptions for identifying the LATE

- Nevertheless, it is still possible to identify the LATE under certain assumptions.
- We utilize the exogeneity of Z_i, which is also known as an instrumental variable (IV) or instrument.
- Assumption 1 (random assignment):

$$Z_i \perp \{Y_i(1), Y_i(0)\},$$
$$\varepsilon < P(Z_i = 1) < 1 - \varepsilon.$$

- This is guaranteed by experimental design and not testable.

Assumptions for identifying the LATE

- Assumption 2 (exclusion restriction):

$$Y_i(D_i, Z_i) = Y_i(D_i).$$

- It means that Z_i affects the outcome only through D_i.
- In theory, Y_i's value could be affected by Z_i.
- E.g., being in the treatment group makes you feel optimistic about your health.
- Or, being in the treatment group makes you less cautious in your daily life.
- It is not testable as well.
- But we can make it more plausible via better designs (e.g., double-blind with a placebo treatment).
- More problematic in observational studies.

Assumptions for identifying the LATE

- Assumption 3 (first stage):

$$Cov(Z_i, D_i) \neq 0.$$

- It means that the assignment does affect the actual treatment status.
- We can test it by checking the correlation between Z_i and D_i.
- The correlation reflects the proportion of individuals whose treatment status are affected by the assignment (compliers and defiers).
- An instrument is weak if the correlation coefficient is sufficiently small.
- Weak instruments lead to suspicious estimates.

Assumptions for identifying the LATE

- Assumption 4 (monotonicity): $D_i(1) \geq D_i(0)$.
- It excludes the existence of defiers from the sample.
- It seems natural in many scenarios but not so in others.
- But there are mavericks in the world...
- Assumptions 3 and 4 suggest that there is a non-negligible set of compliers.
- We can test exclusion restriction and monotonicity together (later).
- If the other direction is true, we just need to redefine the treatment.

Identify LATE

- Under motononicity, we have

Values	$D_i = 0$	$D_i = 1$
$Z_i = 0$	Never-taker/Complier	Always-taker
$Z_i = 1$	Never-taker	Always-taker/Complier

- Therefore, we can calculate the proportion of never-takers ($q_n = P(D_i = 0|Z_i = 1)$) and always-takers ($q_a = P(D_i = 1|Z_i = 0)$).

Identify LATE

- From random assignment, we know that q_n, q_a, q_c are roughly the same in the treatment group and the control group.
- Knowing q_n and q_a, it is easy to calculate q_c:

$$\begin{aligned} q_c &= P(D_i = 1|Z_i = 1) - P(D_i = 1|Z_i = 0) \\ &= P(D_i = 0|Z_i = 0) - P(D_i = 0|Z_i = 1) \\ &= E[D_i|Z_i = 1] - E[D_i|Z_i = 0] \end{aligned}$$

Identify LATE

- Suppose we have 200 individuals in an experiment, 100 treated and 100 under control:

Values	$D_i = 0$ (110)	$D_i = 1$ (90)
$Z_i = 0$ (100)	80	20
$Z_i = 1$ (100)	30	70

- We can see that $q_n = 30/100 = 0.3$, $q_a = 20/100 = 0.2$.
- We also know that $q_n + q_c = 0.8$, $q_a + q_c = 0.7$.
- Obviously, $q_c = 0.5$.

Identify LATE

- Exclusion restriction indicates that all the effects on Y_i are caused by D_i rather than Z_i.
- We know the total effect of the treatment is $\tau_{ITT} = E[Y_i|Z_i = 1] - E[Y_i|Z_i = 0]$.
- The effect on the compliers, or the LATE, equals to

$$\tau_{LATE} = \frac{\tau_{ITT}}{q_c} = \frac{E[Y_i|Z_i = 1] - E[Y_i|Z_i = 0]}{E[D_i|Z_i = 1] - E[D_i|Z_i = 0]}.$$

- The assumption of first stage ensures that the denominator is non-zero.

Estimate LATE

- In practice, we estimate the LATE using its sample analogue:

$$\hat{\tau}_{Wald} = \frac{\frac{1}{N_1}\sum_{i=1}^{N} Y_i Z_i - \frac{1}{N_0}\sum_{i=1}^{N} Y_i(1-Z_i)}{\frac{1}{N_1}\sum_{i=1}^{N} D_i Z_i - \frac{1}{N_0}\sum_{i=1}^{N} D_i(1-Z_i)}.$$

- This is called the Wald estimator.
- It is the ratio of two estimates.
- Hence, the Wald estimator is biased.

Inference on LATE

- We already know the property of each Hajek estimator.
- It is consistent and and asymptotically normal.
- We can similarly derive the distribution of the Wald estimator through linearization.
- Let $\hat{\tau}_Y = \frac{1}{N_1}\sum_{i=1}^{N} Y_i Z_i - \frac{1}{N_0}\sum_{i=1}^{N} Y_i(1-Z_i)$ and $\hat{\tau}_D = \frac{1}{N_1}\sum_{i=1}^{N} D_i Z_i - \frac{1}{N_0}\sum_{i=1}^{N} D_i(1-Z_i)$.
- We know that

$$\sqrt{N}(\hat{\tau}_Y - \tau_{ITT}) \to \mathcal{N}(0, \sigma_Y^2)$$
$$\sqrt{N}(\hat{\tau}_D - q_c) \to \mathcal{N}(0, \sigma_D^2)$$

Inference on LATE

- Then,

$$\begin{aligned}\sqrt{N}(\hat{\tau}_{Wald} - \tau_{LATE}) &= \sqrt{N}\left(\frac{\hat{\tau}_Y}{\hat{\tau}_D} - \frac{\tau_{ITT}}{q_c}\right) \\ &\approx \frac{\sqrt{N}(\hat{\tau}_Y - \tau_{ITT})}{q_c} - \frac{\tau_{ITT}\sqrt{N}(\hat{\tau}_D - q_c)}{q_c^2} \\ &\to \mathcal{N}(0, \sigma^2_{Wald})\end{aligned}$$

- The weighted average of two normal distributions is still a normal distribution.
- Hence, $\hat{\tau}_{Wald}$ is consistent and asymptotically normal.

Application

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## The LATE is 1.999

## Estimate from the Wald estimator is 1.987
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Better LATE than nothing

- Econometricians often criticize LATE for not capturing any "deep parameters" (Deaton 2009; Heckman and Urzua 2010).
- The values of principal strata effects vary across experiments.
- Nevertheless, LATE is the best we can obtain without more structural assumptions.
- We can generalize the LATE to the ATE by modeling the non-compliance behavior.
- It is always important to understand the effect on those whose behavior will be affected by the policy (Imbens 2010).

References I

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